Adaptive Fuzzy Generalized Predictive Control Based on Discrete-Time T-S Fuzzy Model

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Abstract

The paper presents an adaptive fuzzy predictive control based on discrete-time Takagi-Sugeno (T-S) fuzzy model. The proposed controller is based on Generalized predictive control (GPC) algorithm, and a discrete-time T-S fuzzy model is employed to approximate the unknown nonlinear process. To provide a better accuracy in identification of unknown parameters of the model, it is proposed an on-line adaptive law which ensures that the tracking error remains bounded. The stability of closed-loop control system is proved/studied via the Lyapunov stability theory. To validate the theoretical developments and to demonstrate the performance of the proposed control is simulated as nonlinear system a laboratory-scale liquid-level process. The simulation results show that the proposed method has a good performance and disturbance rejection capacity in industrial process.

Keywords: Generalized predictive control, Fuzzy Control, Adaptive Control, T-S fuzzy, Lyapunov Stability.

1 Introduction

Generalized predictive control (GPC) [3] has become one of the most popular and powerful control methods in industry. It is a model-based control method where a plant model is used to obtain a predictor model. The GPC has been applied in various plants, and has shown good performance results [5], [15]. However the most plants where GPC has been applied were linear because the quadratic optimization problem involved in GPC is easily solved for the linear prediction case. Previous research has presented GPC for nonlinear plants [12], [21]. The first approach was the linearisation of the model plants [12], [21]. However, this approach may not predict exactly because the operating point may change and the predictor does not remain valid. The disadvantage of GPC, as common factor of all Model Based Predictive Control (MBPC) is its assumption of an accurate model.

This assumption may present problems, because many complex plants are difficult to be modelled mathematically based in physical laws, or have large uncertainties and strong nonlinearities. An alternative to modelling nonlinear plants are fuzzy logic systems. Fuzzy systems may be used to approximate unknown nonlinear functions of the plant. This is theoretically supported by the fact that fuzzy logic systems are universal approximators [17], [7]. Takagi-Sugeno (T-S) [14] fuzzy models have gained much popularity because of their rule consequent structure. The main difference between T-S fuzzy models and other fuzzy models is that the consequent of a T-S fuzzy model is a real-valued function. In off-line training algorithms the discrete-time T-S fuzzy model can be obtained from input-output data collected from a plant. However, this collected dataset can be limited and the obtained T-S fuzzy models may not provide adequate accuracy. This motivates the introduction of adaptive control methodologies to solve the problem.

Adaptive systems are generally used to control structures whose parameters are unknown and/or time-varying. Adaptive fuzzy controllers can be classified into two categories [16]: direct and indirect adaptive controllers. In direct adaptive fuzzy control, the parameters of the controller initially constructed from human control knowledge, and the iteratively adjusted to reduce the output error between the plant and a reference model. Indirect adaptive fuzzy control, are initially constructed from human knowledge about the unknown plant, and then iteratively adjusted to reduce the output error between the plant and a reference model. This paper focuses on indirect adaptive control, where a T-S fuzzy model is adapted on-line.

In [18] a control strategy for plants with multiple time-delay of state variables and manipulated variables is proposed and simulated in a truck-trailer system. In [13] an adaptive predictive control method based on T-S fuzzy models is proposed for discrete-time nonlinear systems, where the consequent parameters of T-S fuzzy model are identified by a weighted recursive least squares method. In [9] a Locomotive Brake Control Method based on T-S Fuzzy Modeling Predictive Control is proposed. A fuzzy clustering method is used to determine initial parameters, and a back-propagation algorithm used for parameters adaptation by off-line learning. In [19] a robust fuzzy model predictive control method using uncertain T-S fuzzy systems is proposed for discrete-time nonlinear plants subject to actuator saturation. However, the above methods need some knowledge about the system to be controlled. Such knowledge can be difficult to extract in complex industrial processes. In [2] and [4] are used neural networks as empirical models in model predictive control. However, the use of neural network is computationally demanding due to the on-line optimization required to compute the control signals.

In this paper, a new adaptive fuzzy model-based predictive controller is proposed for a class of nonlinear discrete-time processes. The proposed controller is based on GPC algorithm and uses an T-S fuzzy model adapted on-line, and is able to ensure that the tracking error remains bounded. The stability of closed-loop control system is proved/studied via the Lyapunov stability theory.

A diagram of the proposed adaptive fuzzy generalized predictive control (AFGPC) approach is represented in Fig. 1. As can be seen, the control scheme consists of the plant, the controller, and the adaptive T-S fuzzy model. The proposed controller, is composed by a model-based predictive control whose model parameters are adjusted on-line by an adaptation law based in Lyapunov Theory. In the presently proposed method, human knowledge about the plant model is not necessary.

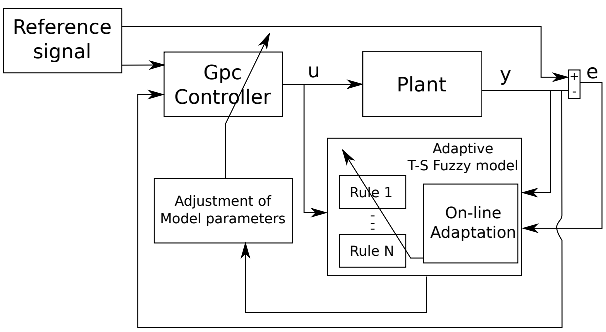

Figure 1: A generic schematic diagram of the proposed control AFGPC.

The paper is organized as follows. Section 2 presents a nonlinear systems modelling method using T-S fuzzy models. Section 3 presents a brief overview of GPC. The proposed control method is described in Section 4. In Section 5, the results of simulation are presented and analysed. Finally, Section 6 makes concluding remarks.

2 Nonlinear Systems Modelling Using T-S Fuzzy Models

Takagi-Sugeno (T-S) fuzzy models with the simplified linear rule consequent are universal approximators capable of approximating any continuous nonlinear system [20]. A large class of nonlinear processes can be represented by the following NARMAX model [8]:

$$\begin{aligned} y(k) = f[&y(k-1), y(k-2), \ldots, y(k-n_y), \\ &u(k-d-1), \ldots, u(k-d-n_u)] + \zeta(k). \end{aligned} \qquad (1)$$

where $u(\cdot) : \mathbb{N} \to \mathbb{R}$ and $y(\cdot) : \mathbb{N} \to \mathbb{R}$ are the process input and output, $f(\cdot) : \mathbb{R}^{n_y+n_u+d+1} \to \mathbb{R}$ represents a nonlinear mapping which is assumed to be unknown, $n_u \in \mathbb{N}$ and $n_y \in \mathbb{N}$ are the orders of input and output respectively, $d \in \mathbb{N}$ is the time-delay of the system, and $\zeta(k) \in \mathbb{R}$ is a sequence of zero-mean Gaussian white noise. Since $f(\cdot)$ in the discrete-time nonlinear SISO plant (1) is unknown, then $f(\cdot)$ will be approximated by a T-S fuzzy system. To design the T-S fuzzy model, the global operation of the nonlinear system (1) can be accurately approximated into several local affine models. Thus, system (1) can be described by a T-S fuzzy model defined by the following fuzzy rules:

$$\begin{aligned} R_i :\quad & \text{IF } x_1(k) \text{ is } A_1^i, \text{ and} \ldots \text{and } x_n(k) \text{ is } A_n^j \\ & \text{THEN } y_i(k) = a_i(z^{-1})y(k-1) + \\ & \qquad b_i(z^{-1})u(k-d-1) + \zeta(k), \\ & i = 1, \ldots, N, \end{aligned} \qquad (2)$$

where R_i $(i = 1, 2, \ldots, N)$ represents the i-th fuzzy rule, N is the number of rules,

$$\begin{aligned} a_i(z^{-1}) &= a_{1i} + a_{2i}z^{-1} + \ldots + a_{n_y i}z^{-(n_y-1)}, \\ b_i(z^{-1}) &= b_{0i} + b_{1i}z^{-1} + \ldots + b_{n_u i}z^{-n_u}, \end{aligned} \qquad (3)$$

and $u(k)$ is the control output. $x_1(k), \ldots, x_n(k)$ are the input variables of the T-S fuzzy system - they can be any variables chosen by the designer [e.g. $y(k-1)$, $u(k-1)$, or other]. A_i^j are linguistic terms characterized by fuzzy membership functions $\mu_{A_i^j}(x_i)$ which describe the local operating regions of the plant. In the sequel, deterministic models will be considered [$\zeta(k) = 0$]. Thus, $y(k)$ can be rewritten as

$$\begin{aligned} y(k) &= \sum_{i=1}^{N} \bar{\omega}^i(\mathbf{x}(k))\theta^{i^T}\mathbf{x}_e(k)^T, \\ &= \mathbf{\Theta}^T\mathbf{\Psi}(\mathbf{k}), \end{aligned} \qquad (4)$$

where, for $i = 1, \ldots, n$,

$$\begin{aligned} \mathbf{x}(k) &= [x_1(k), x_2(k), \ldots, x_n(k)]^T, & (5) \\ \bar{\omega}^i[\mathbf{x}(k)] &= \frac{\prod_{j=1}^{n} A_j^i(x_j)}{\sum_{i=1}^{N}\prod_{j=1}^{n} A_j^i(x_j)}, \\ \theta^i &= \left[a_{1i}, \ldots, a_{n_y i}, b_{1i}, \ldots, b_{1n_u}\right]^T, & (6) \\ \mathbf{\Theta} &= \left[\theta^{1^T}, \theta^{2^T}, \ldots, \theta^{N^T}\right]^T, & (7) \\ \mathbf{x}_e(k) &= [y(k-1), \ldots, y(k-n_y), \\ &\quad u(k-d-1), \ldots, u(k-d-n_u)]^T, \\ \mathbf{\Psi}^T(k) &= \left[\left(\bar{\omega}^1[\mathbf{x}(k)]\right)\mathbf{x}_e^T(k), \ldots,\right. \\ &\quad \left.\left(\bar{\omega}^N[\mathbf{x}(k)]\right)\mathbf{x}_e^T(k)\right]^T. \end{aligned}$$

Assumption 1. [11] There exists a (optimal) model parameter vector $\mathbf{\Theta}_*$ in T-S fuzzy model that makes (4) become a perfect representation of the real plant (1).

Taking into account Assumption 1, i.e. assuming there is no modelling error, and using (4), then the real plant (1) can be is represented as

$$y(k) = \mathbf{\Theta}_*^T \mathbf{\Psi}(k), \tag{8}$$

where $\mathbf{\Theta}_* = \left[\theta_*^{1^T}, \theta_*^{2^T}, \ldots, \theta_*^{N^T}\right]^T$. It is assumed that the parameters vector $\mathbf{\Theta}_*$ in (8) is unknown. Thus, an approximate model for $y(k)$ is defined as

$$\begin{aligned} \hat{y}(k) &= \sum_{i=1}^{N} \bar{\omega}^i(\hat{\mathbf{x}}(k)) \hat{\theta}^{i^T} \hat{\mathbf{x}}_e(k)^T, \\ &= \hat{\mathbf{\Theta}}^T(k) \mathbf{\Psi}(k), \end{aligned} \tag{9}$$

where $\hat{\mathbf{\Theta}} = \left[\hat{\theta}^{1^T}, \hat{\theta}^{2^T}, \ldots, \hat{\theta}^{N^T}\right]^T$ is vector of adjustable parameters which is an estimate of $\mathbf{\Theta}_*$ and $\hat{\theta}^i = \left[\hat{a}_{1i}, \ldots, \hat{a}_{n_y i}, \hat{b}_{1i}, \ldots, \hat{b}_{1n_u}\right]^T$.

3 Predictive Control Law

The adaptive fuzzy generalized predictive control (AFGPC) developed in this paper is motivated from the GPC strategy [3]. For completeness this section briefly overviews the GPC. It is assumed that the plant model is of the form (4), which can be rewritten as follows [6]:

$$\bar{a}(z^{-1}) y(k) = \bar{b}(z^{-1}) u(k-d-1) + \zeta(k), \tag{10}$$

where

$$\begin{aligned} \bar{a}(z^{-1}) &= 1 - \bar{a}_1(z^{-1}) - \ldots - \bar{a}_{n_y}(z^{-n_y}), && (11) \\ \bar{b}(z^{-1}) &= \bar{b}_1(z^{-1}) + \ldots + \bar{b}_{n_u}(z^{-n_u}), && (12) \\ \bar{a}_j &= \sum_{i=1}^{N} \bar{\omega}^i(\mathbf{x}(k)) a_{ji}(z^{-j}), && (13) \\ \bar{b}_j &= \sum_{i=1}^{N} \bar{\omega}^i(\mathbf{x}(k)) b_{ji}(z^{-j}). && (14) \end{aligned}$$

The GPC the control laws is obtained to minimize the following cost function

$$\begin{aligned} J(k) = &\sum_{p=d}^{N_p} \left[\hat{y}(k+p|k) - \phi_p r(k+p)\right]^2 \\ &+ \sum_{p=d}^{d+N_u-1} \left[q(z^{-1}) \Delta u(k+p-d|k)\right]^2, \end{aligned} \tag{15}$$

where $\hat{y}(k+p|k)$ is an optimum p-step ahead prediction of the system on instant k, $r(k+p)$ is the future reference trajectory, ϕ_p is the feed forward gain, $\Delta = 1 - z^{-1}$, and $q(z^{-1}) = q_0 + q_1 z^{-1} + \ldots + q_{N_p+n_u-1} z^{-(N_p+n_u-1)}$ is a weighting polynomial. N_p and N_u are output and control horizons, respectively. Consider the following Diophantine equation (16):

$$\begin{aligned} 1 &= \Delta e_p(z^{-1}) \bar{a}(z^{-1}) + z^{-p} f_p(z^{-1}), && (16) \\ e_p(z^{-1}) &= 1 + e_{p,1} z^{-1} + \ldots + e_{p,p-1} z^{-(p-1)}, && (17) \\ f_p(z^{-1}) &= f_{p,0} + f_{p,1} z^{-1} + \ldots + f_{p,n_y} z^{-n_y}, && (18) \end{aligned}$$

where $e_p(z^{-1})$ and $f_p(z^{-1})$ are polynomials obtained by [3]. Multiplying (10) by $\Delta z^p e_p(z^{-1})$ yields

$$\begin{aligned} &\Delta z^p e_p(z^{-1}) \bar{a}(z^{-1}) y(k) = \\ &\Delta z^p e_p(z^{-1}) \bar{b}(z^{-1}) u(k-d-1) + \Delta z^p e_p(z^{-1}) \zeta(k). \end{aligned} \tag{19}$$

Defining

$$\begin{aligned} \hat{\zeta}(k) &= \Delta z^p e_p(z^{-1}) \zeta(k), && (20) \\ g_p(z^{-1}) &= \Delta e_p(z^{-1}) \bar{b}(z^{-1}), && (21) \\ g_p(z^{-1}) &= g_{p,0} + g_{p,1} z^{-1} + \ldots + \\ &\quad\; g_{p,p+n_u-1} z^{-(p+n_u-1)}. \end{aligned}$$

and substituting (16), (20)-(21) in (19) yields

$$\begin{aligned} y(k+p|k) &= f_p(z^{-1}) y(k) + \\ &\quad\; g_p(z^{-1}) \Delta u(k+p-d-1) + \hat{\zeta}(k). \end{aligned} \tag{22}$$

Thus, the best prediction of $y(k+p|k)$ is

$$\begin{aligned} \hat{y}(k+p|k) &= f_p(z^{-1}) y(k) + \\ &\quad\; g_p(z^{-1}) \Delta u(k+p-d-1). \end{aligned} \tag{23}$$

To reduce computation costs $N_u = 1$ is chosen. Assuming $\Delta u(k+1) = \ldots = \Delta u(k+N_p) = 0$ and using (24), then (15) can be rewritten as

$$\begin{aligned} J_{eq}(k) = &\left[\mathbf{F} y(k) + \mathbf{G} \Delta u(k) + \mathbf{L} - \mathbf{\Phi R}\right]^T \left[\mathbf{F} y(k)\right. \\ &\left. + \mathbf{G} \Delta u(k) + \mathbf{L} - \mathbf{\Phi R}\right] + \left[q(z^{-1}) \Delta u(k)\right]^2, \end{aligned} \tag{24}$$

where

$$\begin{aligned} \mathbf{F} &= \left[f_d(z^{-1}), \ldots, f_{N_p}(z^{-1})\right]^T, && (25) \\ \mathbf{G} &= \left[g_{d,0}, g_{d+1,0}, \ldots, g_{N_p,0}\right], && (26) \\ \mathbf{L} &= \left[\sum_{\rho=1}^{d+n_u-1} g_{d,\rho} \Delta u(k-\rho), \right. \\ &\quad\; \sum_{\rho=1}^{d+n_u} g_{d+1,\rho} \Delta u(k-\rho), \\ &\quad\; \left. \ldots, \sum_{\rho=1}^{N_p+n_u-1} g_{N_p,\rho} \Delta u(k-\rho)\right]^T, && (27) \\ \mathbf{\Phi} &= \mathrm{diag}\left\{\phi_d, \phi_{d+1}, \ldots, \phi_{N_p}\right\}, && (28) \\ \mathbf{R} &= \left[r(k+d), \ldots, r(k+N_p)\right]^T. && (29) \end{aligned}$$

To minimize $J_{eq}(k)$ the following equation is solved

$$\frac{\partial J_{eq}(k)}{\partial[\Delta u(k)]} = 0. \tag{30}$$

By minimizing $J_{eq}(k)$ using (30), the following optimum control increment is obtained (see [10]):

$$\Delta u(k) = \frac{\mathbf{G}^T(\mathbf{\Phi}\mathbf{R} - \mathbf{F}y(k))}{\mathbf{G}^T\mathbf{G} + \lambda}, \lambda = q_0^2 > 0. \tag{31}$$

4 Adaptive Predictive Fuzzy Control

This section explains how to formulate the T-S fuzzy model (Sections 2) that will be used in the predictive control law (Section 3) such that their model parameters can be adapted in novel Adaptive Predictive Fuzzy Control framework. Taking into account a nonlinear discrete-time dynamic system model and the predictive control law, the closed-loop dynamic error equation will be determined. The next step will be to choose an adaptive law to minimize the tracking error **e** and the parameters error $\tilde{\mathbf{\Theta}}$ by the minimization of a candidate Lyapunov function.

To design the adaptive predictive fuzzy control architecture consider a class of nonlinear discrete-time dynamic systems modelled by:

$$\begin{aligned} x_n(k+1) &= f[\mathbf{x}(k)] + g[\mathbf{x}(k)]u(k), \\ y(k+1) &= x_n(k+1), \end{aligned} \tag{32}$$

where $\mathbf{x}(k) \hat{=} [x_1(k), x_2(k), \ldots, x_n(k)]$ is the state vector, u is the control input, y is the output of the system, and $f[\mathbf{x}(k)]$ and $g[\mathbf{x}(k)]$ are unknown functions.

Assumption 2. [11] $|g(\mathbf{x}(k))| > \epsilon$, where ϵ is a small real positive number, which implies that the relative degree of the T-S fuzzy model and, consequently, the relative degree of the plant are both equal to one.

Without loss of generality, it is assumed that $g[\mathbf{x}(k)] > 0$. To simplify the process of computer calculation it is considered that $g[\mathbf{x}(k)] = g > 0$ is constant.

Assumption 3. [11] The reference trajectory $r(k)$ satisfies

$$\|r(k)\| \leq U, \tag{33}$$

where U is a known bound.

For the moment, assume that functions $f[\mathbf{x}(k)]$ and $g[\mathbf{x}(k)] = g$ are known. Let $\mathbf{k} = [k_n, \ldots, k_1]^T$ be chosen such that the zeros of polynomial $k_z = z^n + k_1 z^{n-1} + \ldots + k_n$ are inside in the unit circle centered at the origin of the z plane, and chose the control law

$$u_*(k) = \frac{1}{g}\left\{-f[\mathbf{x}(k)] + r(k) + \mathbf{k}^T\mathbf{e}(k)\right\}, \tag{34}$$

where $r(k)$ is the reference model output signal, and

$$\begin{aligned} \mathbf{e}(k) &= [e(k-n-1), \ldots, e(k-1), e(k)]^T, && (35) \\ e(k) &= r(k) - y(k). && (36) \end{aligned}$$

Substituting (36) into (32) and after some manipulation with (34), the following closed-loop dynamic equation is obtained:

$$e(k+1) = -\mathbf{k}^T\mathbf{e}(k) + g[u_*(k) - u(k)]. \tag{37}$$

Let

$$\mathbf{\Lambda} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -k_n & -k_{n-1} & \ldots & \ldots & -k_1 \end{bmatrix}, \mathbf{b_g} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ g \end{bmatrix}, \tag{38}$$

then (37) can be rewritten into vector form as

$$\mathbf{e}(k+1) = \mathbf{\Lambda}\mathbf{e}(k) + \mathbf{b_g}[u_*(k) - u(k)]. \tag{39}$$

Assuming that $f[\mathbf{x}(k)]$ is unknown, rewritting (31) as

$$\begin{aligned} u(k) &= \mathbf{K}\left[\mathbf{\Phi}\mathbf{R} - \mathbf{F}y(k)\right] + u(k-1), && (40) \\ \mathbf{K} &= \mathbf{G}^T / \left(\mathbf{G}^T\mathbf{G} + \lambda\right), \end{aligned}$$

and substituting $y(k)$ by its T-S fuzzy approximation $\hat{y}(k)$ (9), then the control law is designed as

$$\begin{aligned} u(k) &= u\left[\mathbf{x}_e(k), \hat{\mathbf{\Theta}}\right], && (41) \\ &= \mathbf{K}\left[\mathbf{\Phi}\mathbf{R} - \mathbf{F}\hat{y}(k)\right] + u(k-1), \\ &= \mathbf{K}\left[\mathbf{\Phi}\mathbf{R} - \mathbf{F}\hat{\mathbf{\Theta}}^T(k)\mathbf{\Psi}(k)\right] + u(k-1). \end{aligned}$$

Define the optimal parameter vector

$$\mathbf{\Theta}^* = \text{argmin}_{\hat{\mathbf{\Theta}} \in \Omega_{\mathbf{\Theta}}}\left[\sup_{\mathbf{x}_e \in \Omega_{\mathbf{x}_e}} \left\| u(\mathbf{x}_e, \hat{\mathbf{\Theta}}) - u_* \right\|\right], \tag{42}$$

where $\Omega_{\mathbf{\Theta}}$ and $\Omega_{\mathbf{x}_e}$ are the sets of admissible values of $\hat{\mathbf{\Theta}}$ and $\mathbf{x}_e$ respectively. Thus, from (42), with a minimum approximation error

$$u_*(k) \approx u^*(k) = u\left[\mathbf{x}_e(k), \mathbf{\Theta}^*\right]. \tag{43}$$

where $u^*(k)$ is defined as the optimal command. By substituting $u^*(k)$ (43) for $u_*(k)$, (39) is rewritten as

$$\begin{aligned} \mathbf{e}(k+1) &= \mathbf{\Lambda}\mathbf{e}(k) + \mathbf{b_g}\left[u^*(k) - u(k)\right], \\ &= \mathbf{\Lambda}\mathbf{e}(k) + \mathbf{b_g}\mathbf{K}\mathbf{F}\tilde{\mathbf{\Theta}}^T(k)\mathbf{\Psi}(k), && (44) \end{aligned}$$

where $\tilde{\mathbf{\Theta}}(k) = \hat{\mathbf{\Theta}}(k) - \mathbf{\Theta}^*$.

Assumption 4. [11] There exist $\alpha > 0$ and a positive-definite symmetric matrix **P** such that for matrix $\mathbf{\Lambda}$ (38),

$$\mathbf{\Lambda}^T\mathbf{P}\mathbf{\Lambda} - \mathbf{P} \leq -\alpha I < 0. \tag{45}$$

Consider the candidate Lyapunov function for system (44),

$$V(k) = \frac{1}{2}\mathbf{e}^T(k)\mathbf{P}\mathbf{e}(k) + \frac{1}{2\gamma}\tilde{\mathbf{\Theta}}^T(k-1)\tilde{\mathbf{\Theta}}(k-1), \tag{46}$$

where γ is a positive constant and **P** is a positive-definite symmetric $n \times n$ matrix.

In order to minimize the tracking error **e** and the parameter error $\tilde{\mathbf{\Theta}}$ equation (46) will be minimized. To decrease $V(k)$ it is necessary ensure that $\Delta V(k) < 0$. $\Delta V(k)$ will be analysed and calculated in (47)-(52). Taking the first time difference of (46) and with some manipulations (48) is deduced. Using (44) and defining $\rho = \mathbf{b_g KF}\tilde{\mathbf{\Theta}}^T(k)\mathbf{\Psi}(k)$, (49) can be obtained. By Assumption 4, (50) can be written. Since **P** is symmetric, then $\rho^T\mathbf{P\Lambda e}(k) = [\mathbf{\Lambda e}(k)]^T\mathbf{P}\rho$ and with some manipulations (51) is derived. Then, with some manipulations (52) is obtained.

$$
\begin{aligned}
& \Delta V(k) = V(k+1) - V(k), && (47)\\
= \; & \frac{1}{2}\mathbf{e}^T(k+1)\mathbf{Pe}(k+1) - \frac{1}{2}\mathbf{e}^T(k)\mathbf{Pe}(k) \\
& -\frac{1}{2\gamma}\left[\tilde{\mathbf{\Theta}}(k) - \tilde{\mathbf{\Theta}}(k-1)\right]^T\left[\tilde{\mathbf{\Theta}}(k) - \tilde{\mathbf{\Theta}}(k-1)\right] \\
& +\frac{1}{\gamma}\tilde{\mathbf{\Theta}}^T(k)\tilde{\mathbf{\Theta}}(k) - \frac{1}{\gamma}\tilde{\mathbf{\Theta}}^T(k)\tilde{\mathbf{\Theta}}(k-1), && (48)\\
= \; & \frac{1}{2}\mathbf{e}^T(k)\left(\mathbf{\Lambda}^T\mathbf{P\Lambda} - \mathbf{P}\right)\mathbf{e}(k) + \rho^T\mathbf{P\Lambda e}(k) \\
& -\frac{1}{2\gamma}\left[\tilde{\mathbf{\Theta}}(k) - \tilde{\mathbf{\Theta}}(k-1)\right]^T\left[\tilde{\mathbf{\Theta}}(k) - \tilde{\mathbf{\Theta}}(k-1)\right] \\
& +\frac{1}{\gamma}\tilde{\mathbf{\Theta}}^T(k)\left[\tilde{\mathbf{\Theta}}(k) - \tilde{\mathbf{\Theta}}(k-1)\right] + \frac{1}{2}\rho^T\mathbf{P}\rho, && (49)\\
\leq \; & -\frac{1}{2}\alpha\mathbf{e}^T(k)\mathbf{e}(k) + \rho^T\mathbf{P\Lambda e}(k) + \frac{1}{2}\rho^T\mathbf{P}\rho \\
& +\frac{1}{\gamma}\tilde{\mathbf{\Theta}}^T(k)\left[\tilde{\mathbf{\Theta}}(k) - \tilde{\mathbf{\Theta}}(k-1)\right], && (50)\\
= \; & -\frac{1}{2}\alpha\mathbf{e}^T(k)\mathbf{e}(k) + \frac{1}{2}\rho^T\mathbf{P}\rho \\
& +\tilde{\mathbf{\Theta}}^T(k)\left[\mathbf{\Lambda e}(k)\right]^T\mathbf{Pb_g KF\Psi}(k) && (51)\\
= \; & -\frac{1}{2}\alpha\mathbf{e}^T(k)\mathbf{e}(k) + \frac{1}{2}\rho^T\mathbf{P}\rho \\
& +\tilde{\mathbf{\Theta}}^T(k)\left\{\left[\mathbf{\Lambda e}(k)\right]^T\mathbf{Pb_g KF\Psi}(k) + \right. && (52)\\
& \left. \frac{1}{\gamma}\left[\tilde{\mathbf{\Theta}}(k) - \tilde{\mathbf{\Theta}}(k-1)\right]\right\}
\end{aligned}
$$

To minimize $V(k)$, the following parameter adaptation law is chosen such that the second term in (52) is zero

$$
\begin{aligned}
& \left[\mathbf{\Lambda e}(k)\right]^T\mathbf{Pb_g KF\Psi}(k) + \frac{1}{\gamma}\left[\tilde{\mathbf{\Theta}}(k) - \tilde{\mathbf{\Theta}}(k-1)\right] = 0, \\
& \hat{\mathbf{\Theta}}(k) = \hat{\mathbf{\Theta}}(k-1) - \gamma\left[\mathbf{\Lambda e}(k)\right]^T\mathbf{Pb_g KF\Psi}(k), \qquad (53)
\end{aligned}
$$

where γ is a gain in adaptive law (53). This adaptive law permits the adaption of T-S fuzzy model parameters (2) which, with some manipulation explained in Secs. 2 and 3, will be used in (10). Finally, as explained in Sec. 3 is obtained the optimum control increment (31).

Using (53), equation (52) can be rewritten as

$$
\begin{aligned}
\Delta V(k) & \leq -\frac{1}{2}\alpha\mathbf{e}^T(k)\mathbf{e}(k) + \frac{1}{2}\rho^T\mathbf{P}\rho \\
& \leq -a\|\mathbf{e}(k)\|^2 + \lambda_{max}(\mathbf{P})\|\rho\|^2, \qquad (54)
\end{aligned}
$$

where $\lambda_{max}(\mathbf{P})$ is the largest singular value of **P** and

$$
\begin{aligned}
\|\rho\| & = \|\mathbf{b_g KF}\tilde{\mathbf{\Theta}}^T(k)\mathbf{\Psi}(k)\|, \\
& \leq \|\mathbf{b_g}\|\,\|\mathbf{KF}\|\,\|\tilde{\mathbf{\Theta}}^T(k)\mathbf{\Psi}(k)\|, \qquad (55)\\
& \leq \rho_c. \qquad (56)
\end{aligned}
$$

From (54) and (56), there exists a positive constant ρ_c such that $\Delta V(k) \leq 0$ outside the ball

$$
\left\{\mathbf{e}(k) : \|\mathbf{e}(k)\| < \epsilon = \sqrt{\frac{\lambda_{max}(\mathbf{P})}{a}\rho_c}\right\}. \qquad (57)
$$

Theorem 1. Consider the closed loop system consisting of the plant (8), controller (31) and parameter adaptation laws (53). If Assumptions 1-4 hold, then the plant tracking error vector $\mathbf{e}(k)$ is bounded above by ϵ defined in (57).

The proof of Theorem 1 is given by the above analysis and by (57).

5 Simulation Results

This section presents simulation results to validate the theoretical developments and to demonstrate the performance of the proposed adaptive predictive fuzzy control scheme in nonlinear systems. In the simulation to test the reference tracking performance, parameters convergence, and disturbance rejection capacity, the reference input $r(k)$ is changed with time and a load disturbance $v(k)$ is applied.

5.1 Control of a Laboratory-Scale Liquid-Level Process

In this simulation, the following nonlinear model of a laboratory-scale liquid-level process is considered [1]:

$$
\begin{aligned}
y(k) = \; & 0.9722y(k-1) + 0.3578u(k-1) \\
& -0.1295u(k-2) - 0.04228y^2(k-2) \\
& -0.3103y(k-1)u(k-1) \\
& +0.1663y(k-2)u(k-2) \\
& -0.03259y^2(k-1)y(k-2) \\
& -0.3513y^2(k-1)u(k-2) \\
& +0.3084y(k-1)y(k-2)u(k-2) \\
& +0.1087y(k-2)u(k-1)u(k-2). \qquad (58)
\end{aligned}
$$

Taking into account (58) the input order, output order, and time-delay are $n_u = 2$, $n_y = 2$ and $d = 1$, respectively. The following controller parameters were chosen by the user by trial and error: $N_p = 15$, $\lambda = 50$, $g = 1$, $k_1 = 0.9$, $\mathbf{P} = 1$, $\gamma = 0.4$. The reference input was

$$
r(k) = \begin{cases} 1, & 0 < k \leq 400, \\ 0.2, & 400 < k \leq 700, \\ 1, & 700 < k \leq 1400, \end{cases} \qquad (59)
$$

and the load disturbance was $v(k) = 0.08$ for $k \geq 900$, and $v(k) = 0$, otherwise. The input variables (5) of the

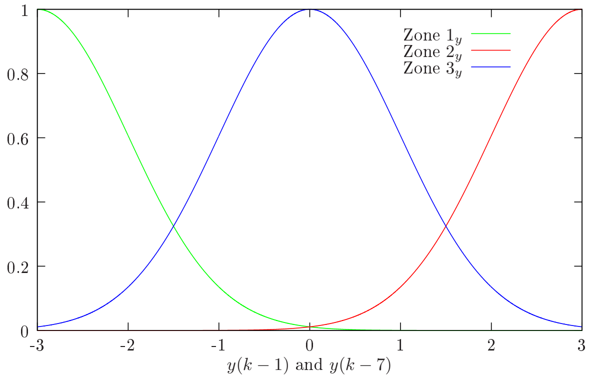

Figure 2: Fuzzy membership functions of input variables $y(k-1)$ and $y(k-7)$.

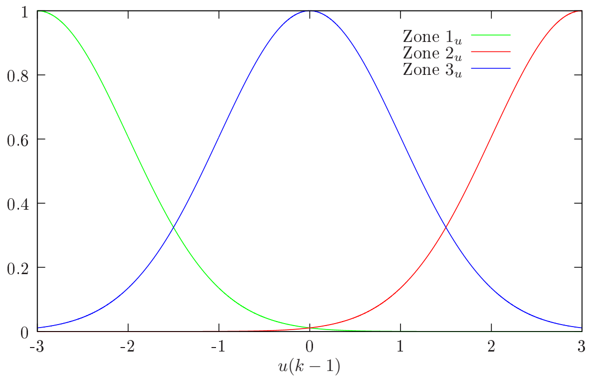

Figure 3: Fuzzy membership functions of input variable $u(k-1)$.

fuzzy rules were chosen as $\mathbf{x}(k) = [y(k-1), y(k-7), u(k-1)]^T$, where to save the computational cost in each input variable there are 3 membership functions that were designed taking into account the corresponding range, $y(k-1), y(k-7) \in [-3;3]$, $u(k-1) \in [-3;3]$, as illustrated in Figs. 2 and 3. The fuzzy rule-base contains rules covering all combinations of membership functions of the 3 input variables, giving a total of $3^3 = 27$ rules. All controller adjustable parameters [consequent parameters of the rules (2), components of (7)], are initialized to 0.01, to represent the initial absence of knowledge about the plant (58).

5.2 Analysis of Results

From the results shown in Figs. 4, 5, 6, and 7 it can be seen that the proposed controller is able to adequately (attain and) control the system output at the desired reference $r(k)$.

In terms of initial response of the controller, it can be observed that although there is no initial model knowledge (parameters initialized 0.01), the controller quickly reaches the desired reference signal.

When the load disturbance $v(k)$ is applied at $k = 900$,

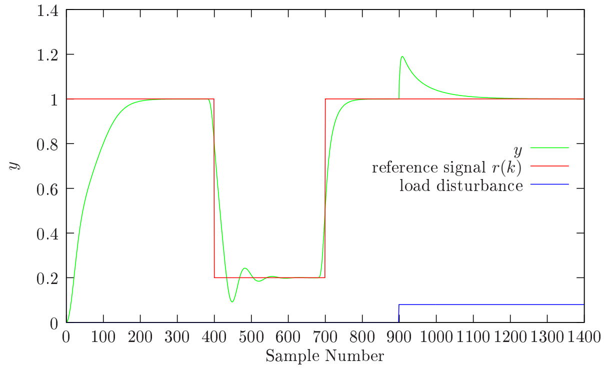

Figure 4: Results of the proposed controller in presence of load disturbances.

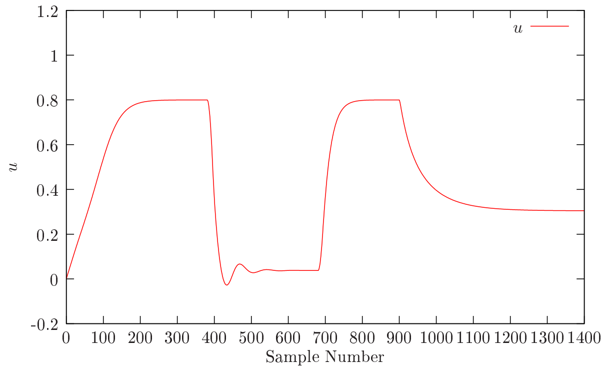

Figure 5: Applied command signal.

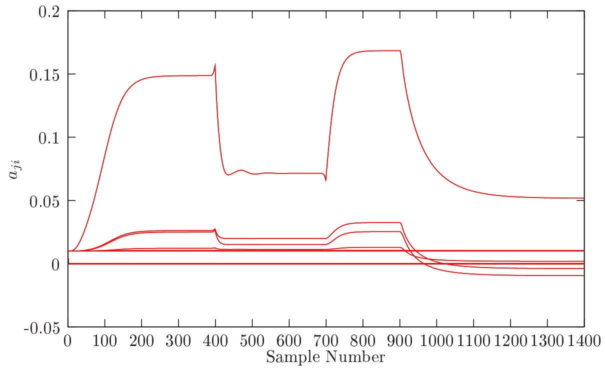

Figure 6: Temporal evolution of the adjustable parameters a_{1i} and a_{2i}.

there is an overshoot in system response. As can be seen the controller eliminates this disturbance.

The temporal evolution of the adjustable parameters of the controller is show Figs. 6 and 7. The model parameters are initialized with values near zero, but then they are adjusted taking in account the desired response. When the load disturbance is applied, the parameters are again adjusted taking into account the corresponding changes

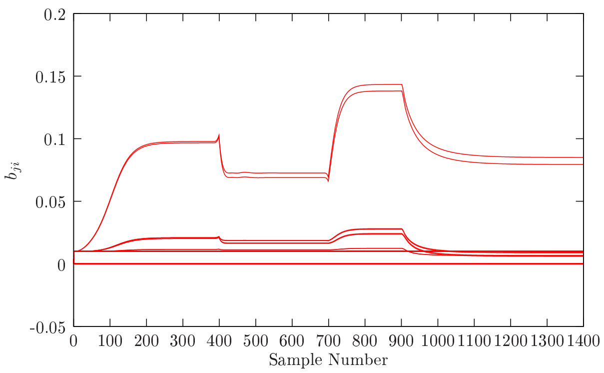

Figure 7: Temporal evolution of the adjustable parameters b_{1i} and b_{2i}.

in the system. Note that some parameters remain constant during the tests. These parameters belong to rules that do not have effect for the specific values that the relevant process variables take during the presented experiments. This illustrates that the adaptation mechanisms worked adequately.

6 Conclusion

This paper has proposed a new adaptive model-based predictive controller for a class of nonlinear discrete-time process. The proposed controller is based on GPC the algorithm and uses an adaptive T-S fuzzy model on-line. If is assumed that initial human knowledge about the plant is almost inexistent. The simulation results show that the proposed method is able to adequately control the plant without human knowledge about the plant model, and has good tracking performance and disturbance rejection capacity. This evidence suggests that the proposed controller could be a good option for industrial process control. As can be seen in the simulations, the adjustable parameters are adjusted for control of the unknown plant and taking into account changes in the system.

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References

[1] M. Ahmed. Neural-net-based direct adaptive control for a class of nonlinear plants. *IEEE Transactions on Automatic Control*, 45(1):119–124, 2000.

[2] J. Atuonwu, Y. Cao, G. Rangaiah, and M. Tade. Nonlinear model predictive control of a multistage evaporator system using recurrent neural networks. *IEEE Conference on Industrial Electronics and Applications*, pages 1662 – 1667, 2009.

[3] E. Camacho and C. Bordons. *Model Predictive Control*. Springer-Verlag, February 1998.

[4] J.-Z. Chu, P.-F. Tsai, W.-Y. Tsai, S.-S. Jang, S.-S. Shieh, P.-H. Lin, and S.-J. Jiang. Multistep model mredictive control based on artificial neural networks. *Industrial & engineering chemistry research*, 42(21):5215–5228, 2003.

[5] D. Clarke. Application of generalized predictive control to industrial process. *IEEE Control Systems Magazine*, 8(2):49–55, 1998.

[6] C.-C. Hsiao, S.-F. Su, T.-T. Lee, and C.-C. Chuang. Hybrid compensation control for affine tsk fuzzy control systems. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 34(4):1865–1873, 2004.

[7] B. Kosko. Fuzzy systems as universal approximators. *IEEE Transactions on Computers*, 43(11):1329–1333, 1994.

[8] M. Lazar and O. Pastravanu. A neural predictive controller for non-linear systems. In *Mathematics and Computers in Simulation*, pages 315–324, 2002.

[9] J. Liu, Z. Huang, W. Liu, Y. Yang, and H. Tong. Locomotive brake control method based on t-s fuzzy modeling predictive control. In *Proceedings of the 2008 Second International Symposium on Intelligent Information Technology Application*, pages 602–607, Washington, DC, USA, 2008. IEEE Computer Society.

[10] C.-H. Lu and C.-C. Tsai. Generalized predictive control using recurrent fuzzy neural networks for industrial processes. *Journal of Process Control*, 17(1):83–82, January 2007.

[11] R. Qi and M. A. Brdys. Stable indirect adaptive control based on discrete-time t-s fuzzy model. *Fuzzy Sets Systems*, 159(8):900–925, 2008.

[12] J. Rossiter, B. Kouvaritakis, and R. Dunnett. Application of generalised predictive control to a boiler-turbine unit for electricity generation. *IEE Proceedings Part D*, 138(1):59–67, 1991.

[13] C. Su, P. Li, and S. Wang. A direct adaptive predictive functional control based on t-s fuzzy model. In *Proceedings of the Sixth International Conference on Intelligent Systems Design and Applications*, pages 92–97, Washington, DC, USA, 2006. IEEE Computer Society.

[14] T. Takagi and M. Sugeno. Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man, and Cybernetics*, 15(1):116–132, February 1985.

[15] M. Tham, F. Vagi, A. Morris, and R. Wood. Multivariable and multirate self-tuning control: a distillation column case study. *IEE Proceedings Part D*, 138(1):9–24, 1991.

[16] L.-X. Wang. *A Course in Fuzzy Systems and Control*. Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 1997.

[17] L.-X. Wang and J. Mendel. Fuzzy basis functions, universal approximation, and orthogonal least-squares learning. *IEEE Transactions on Neural Networks*, 3(5):807–814, September 1992.

[18] S. Wang, Y. Wang, and L. Zhang. Time-delay dependent state feedback fuzzy-predictive control of time-delay t-s fuzzy model. In *Proceedings of the 2008 Fifth International Conference on Fuzzy Systems and Knowledge Discovery*, pages 129–133, Washington, DC, USA, 2008. IEEE Computer Society.

[19] J. wei Wen and F. Liu. Robust model predictive control for fuzzy systems subject to actuator saturation. *Sixth International Conference on Fuzzy Systems and Knowledge Discovery*, 6:400–404, 2009.

[20] H. Ying. General miso takagi-sugeno fuzzy systems with simplified linear rule consequent as universal approximators for control and modeling applications. *IEEE International Conference on Systems, Man, and Cybernetics*, 2:1335–1340, 1997.

[21] Q. Zhu, K. Warwick, and J. Douce. Adaptive general predictive controller for nonlinear systems. *IEE Proceedings Part D*, 138(1):33–40, 1991.