Screw-Based Modeling of Soft Manipulators with Tendon and Fluidic Actuation

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A screw-based formulation of the kinematics, differential kinematics and statics of soft manipulators is presented, which introduces the soft robotics counterpart to the fundamental geometric theory of robotics developed since Brockett’s original work on the subject. As far as the actuation is concerned, the embedded tendon and fluidic actuation are modeled within the same screw-based framework and the screw-system to which they belong is shown. Furthermore, the active and passive motion subspaces are clearly differentiated and guidelines for the manipulable and force-closure conditions are developed. Finally, the model is validated through experiments using the soft manipulator for minimally-invasive surgery STIFF-FLOP.

1 INTRODUCTION

The great majority of the modern literature on modeling and control of continuum/soft manipulators is based somehow on the Piecewise-Constant Curvature (PCC) approach [1], [2]. This approach aims to reduce the virtually infinite number of Degrees of Freedom (DOFs) of a soft manipulator to a finite number of DOFs by assuming a constant curvature for each section. This method has been successfully applied to kinematics modeling [3], [4] dynamics modeling [5], [6], design [7], [8] and control [9], [10] of continuum and soft manipulators, to name just a few. Nevertheless, all those significant achievements inherit the underlying limitations of the PCC approach, which are mainly due to two facts: first, it does not allow torsional deformation which is fundamental to cope with non-negligible external loads; second, the PCC parameterization of the soft manipulator is not based on intrinsic variables of the arm, leading to singularities in the forward kinematics, intermediate maps and analytic Jacobian as opposed to geometric Jacobian.

To overcome those limitations, a new approach has been introduced recently, called Piecewise-Constant Strain (PCS) [11]. In the PCS model a soft manipulator is parametrized by a finite set of strain values, including torsion. These strain values are considered constant along the different sections of the manipulator. With respect to the PCC model, the PCS model not only takes into account shears and torsion deformation, but also, due to its intrinsic parameterization, shows a uniform geometric structure based on screw theory (the study of the Special Euclidean group $SE(3)$ and the corresponding Lie Algebra $se(3)$). This turns out to be, as described in this work, a generalization for soft robotics of the fundamental geometric theory of robotics developed since Brockett’s original work on the subject [12].

In this work, the PCS model is formulated exploiting its geometric structure and, for the first time in literature, the product of exponentials formula and the geometric Jacobian for soft robotics are presented. The geometric Jacobian
is then used to extend the PCS model to the multi-section steady-state case.

As far as the actuation is concerned, the most common kind of actuators used in soft manipulation by far are the tendon and fluidic actuation, with different applications (e.g., [13], [14]). In this work a new model based on screw theory for the tendon and fluidic actuation is proposed and the result is applied to the PCS framework. The model allows any path of the tendon or fluidic chamber inside the soft arm. Here the parallel-to-the-midline path is exhaustively studied and its screw geometric structure is detailed. Furthermore, a technique to clearly differentiate between active and passive DOFs is given and applied to study the manipulable and force-closure conditions of soft manipulators.

The new multi-section steady-state model is finally validated through experiments with the state-of-the-art soft manipulator for minimally invasive surgery STIFF-FLOP [14].

2 PIECE-WISE CONSTANT STRAINS MODEL

In this section the kinematics, differential kinematics and statics of the PCS model for a multisection manipulator are developed exploiting its intrinsic geometric structure.

2.1 Kinematics

According to Cosserat beam theory, the configuration of a deformable body with respect to the spatial frame at a certain time is characterized by a position vector \( u(X) \in \mathbb{R}^3 \) and an orientation matrix \( R(X) \in SO(3) \), parameterized by the material abscissa \( X \in [0,L] \) along the robot arm. Thus, the configuration space is defined as a curve \( g(\cdot) : X \mapsto g(X) \in SE(3) \) with \( g = \begin{pmatrix} R & u \\ 0 & 1 \end{pmatrix} \). Then, the strain state of the soft arm is defined, with respect to the body frame, by the vector field along the curve \( g \) given by \( \xi(X) = g^{-1} \frac{\partial g}{\partial X} X = g^{-1} \xi' \in se(3) \), where the hat is the isomorphism between the twist vector representation and the matrix representation of the Lie algebra se(3). The components of this field are specified in the (micro-)body frames as: \( \xi = \begin{pmatrix} k & q \\ 0 & 0 \end{pmatrix} \in se(3) \), \( k = (k^T, q^T)^T \in \mathbb{R}^6 \), where \( q(X) \in \mathbb{R}^3 \) represents the linear strains, and \( k(X) \in \mathbb{R}^3 \) the angular strains. The tilde is the isomorphism between three dimensional vectors and the skew symmetric matrix \( \in so(3) \). The strain twist above gives the following kinematics equation for deformable beam-like body:

\[
g'(X) = g \tilde{\xi} . \tag{1} \]

Now, considering the strain field \( \xi(\cdot) \) to be piece-wise constant along each of the \( N \) sections of the soft arm, we can replace the continuous field with a finite set of \( N \) twist vectors \( \xi_n (n \in \{1,2,...,N\} \text{ Figure 1} ) \) one for each segment of the soft manipulator indicated as \([0,L_1], [L_1,L_2],...[L_{n-1},L_n] \) (with \( L_N = L \)). Under this assumption, equation (1) becomes an homogeneous, linear, matrix differential equation with constant coefficients, which can be analytically solved at any section \( n \) using the matrix exponential method [15].

\[
g(X) = g(L_{n-1}) e^{(X-L_{n-1}) \tilde{\xi}_n} . \tag{2} \]

It turns out that the infinite series of the exponential in (2) can be expressed in a compact way as follows [16]:

\[
g_n(X) := e^{(X-L_{n-1}) \tilde{\xi}_n} = I_4 + (X - L_{n-1}) \tilde{\xi}_n + 1/\theta^2_n ((X - L_{n-1}) \theta_n - \sin ((X - L_{n-1}) \theta_n)) \tilde{\xi}_n^2 + 1/\theta^3_n ((X - L_{n-1}) \theta_n - \sin ((X - L_{n-1}) \theta_n)) \tilde{\xi}_n^3 , \tag{3} \]

where \( \theta_n^2 = k_n^T k_n \). It is easy to show that the rigid transformation \( g(L_{n-1}) \) in (2) can be obtained from \( g(L_{n-2}) \) and the exponential function (3) calculated with respect to \( \tilde{\xi}_{n-1} \). Applying this operation recursively from tip to base, we obtain the product of exponentials formula for soft robots (Figure 1):

\[
g(X) = \prod_{i=1}^{n} e^{(\min(L_i,X)-L_{i-1}) \tilde{\xi}_i} , \tag{4} \]

where \( L_0 = 0 \). As in the case of rigid manipulators [12], the product of exponentials formula (4) allows to parametrized the configuration of the soft manipulator by means of the constant strains \( \xi_i \) which play the role of the joint twists. The only, yet essential, difference is that in the soft robotics case \( \xi_i \) belongs to the whole 6 dimensional Lie algebra \( se(3) \), while, in the rigid counterpart, it belongs to a one-dimensional subalgebra \( \in \mathfrak{e}(3) \), which generates a one-parameter subgroup in \( SE(3) \) through the exponential map.

2.1.1 Piece-wise Inverse Kinematics

The exponential function (3) can be inverted to give an analytic inverse kinematics for each section. The inverse function is called logarithmic function and reads [16] (with \( x = X - L_{n-1} \) in the following):

\[
x \tilde{\xi}_n = \log(g_n(X)) = \\
1/8 \csc^3(\theta_n/2) \sec(\theta_n/2) [(x \theta_n \cos(2x \theta_n) - \sin(x \theta_n))I_4 - (x \theta_n \cos(x \theta_n) + 2x \theta_n \cos(2x \theta_n) - \sin(x \theta_n) - \sin(2x \theta_n))g_n + (2x \theta_n \cos(x \theta_n) + x \theta_n \cos(2x \theta_n) - \sin(x \theta_n) - \sin(2x \theta_n))g_n^2 - (x \theta_n \cos(x \theta_n) - \sin(x \theta_n))g_n^3] , \tag{5} \]

where \( \theta_n \) can be found from \( Tr(g_n(X)) = 2(1 + \cos(x \theta_n)) \) for \(-\pi < x \theta_n < \pi.\)

2.2 Differential Kinematics

The time evolution of the configuration curve \( g(\cdot) \) is represented, with respect to the body frame, by the twist vector
2.3 Multisection Steady-state Equilibrium

The equation of motion of a continuous Cosserat arm has been derived in [20] (together with shell and 3D body) from the extension to continuum media of the Poincaré equations of mechanics by taking a Lagrangian density $\mathcal{L}(\eta) - \mathcal{L}(\xi)$, being $\mathcal{L}$ and $\mathcal{M}$ the densities of kinetic and elastic energy of the Cosserat beam per unit of material length. For the purpose of this paper, only the steady-state equilibrium with respect to the (micro-)body frame is reported here:

$$\mathcal{F}_i + \mathcal{F}_{\eta} + \mathcal{F}_{\xi} + \mathcal{F}_e = 0,$$

where $\mathcal{F}_i(X) = \partial \mathcal{L}/\partial \dot{X} \in \mathbb{R}^6$ is the wrench of internal forces, $\mathcal{F}_{\eta}(X,t) \in \mathbb{R}^6$ is the distributed actuation loads and $\mathcal{F}_e(X) \in \mathbb{R}^6$ is the external wrench of distributed applied forces. Let us specify the angular and linear components of the internal and external wrenches: $\mathcal{F}_i = [M^i_a \ N^i_a]^T$, $\mathcal{F}_{\eta} = [m^i_\eta \ n^i_\eta]^T$, $\mathcal{F}_{\xi} = [m^i_\xi \ n^i_\xi]^T$, where $N_a(X) \in \mathbb{R}^2$ and $M_a(X) \in \mathbb{R}^2$ are the internal force and torque vectors, $n_\eta(X,t) \in \mathbb{R}^3$ and $m_\eta(X,t) \in \mathbb{R}^3$ are the distributed actuation force and torque input, while $n_e(X) \in \mathbb{R}^3$ and $m_e(X) \in \mathbb{R}^3$ are the distributed external force and torque unit for $X$.

Regarding the wrench of internal forces, a linear elastic constitutive model is chosen:

$$\mathcal{F}_i(X) = \Sigma (\xi - \xi^0),$$

where $\Sigma \in \mathbb{R}^{6 \times 6}$ is the constant screw stiffness matrix, equal to (according to the frame as in Figure 1) $\Sigma = \text{diag}(G_l, G_J, E_J, E_A, G_A, G_A)$, $E$ being the Young module and $G$ the shear modulus, $A$ the section area and $J, I$ respectively the bending and torsion second moment of inertia of the micro-solid; $\xi^0 = [0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$ is the reference straight configuration. Finally, as for the external loads, we have considered a distributed load due to gravity and a concentrated/point load due to externally applied loads or contacts:

$$\mathcal{F}_e = \mathcal{M} \mathcal{A}^{-1} \dot{\mathcal{F}}(X,t) + \delta(X-X) \mathcal{F}_p,$$

where $\mathcal{M}(X) \in \mathbb{R}^{6 \times 6}$ is the screw inertia matrix equal to $\mathcal{M} = \text{diag}(I, J, J, A, A, A)$, $\rho$ being the body density.
\( \mathcal{F} = [0 \ 0 \ 0 \ -9.81 \ 0 \ 0]^T \) is the gravity twist with respect to the spatial frame, \( \mathcal{g}_c \) is the map between the spatial frame and the base frame of the soft manipulator, \( \delta(\cdot) \) is the Dirac distribution, \( \mathcal{F}_p \in \mathbb{R}^6 \) is the wrench corresponding to the point load applied at \( X = \bar{X} \).

In order to project the equilibrium (12) into the joint space, we remind that the total power \( W \) delivered by a distributed wrench \( \mathcal{F}(X) \) has to equate the power delivered by a generalized wrench \( (\mathcal{\mu} \in \mathbb{R}^{6N}) \) or, in other words, the constrain forces do not generate any power [21]. This yields the following equivalence: \( W = \int_0^{L_N} \mathcal{F}^T \eta \ dX = \int_0^{L_N} \mathcal{F}^T J \ dX \frac{\xi}{\zeta} = \mathcal{\mu}^T \frac{\xi}{\zeta} \), and consequently:

\[
\mathcal{\mu} = \int_0^{L_N} J^T \ dX.
\] (15)

Thus, according to (15), projecting equation (12) with \( J^T \) and integrating over the interval \([0, L_N]\), leads to the generalized multisection steady-state equation, as follows:

\[
\begin{align*}
&\left[ \int_0^{L_N} J^T \left( \mathcal{F}_i^* + \text{ad}_{\mathcal{g}_c}^* \mathcal{F}_a \right) + J^T \mathcal{F}_a \ dX \right] \\
&\quad + \left[ \int_0^{L_N} J^T \mathcal{M} \text{Ad}_{\mathcal{g}_c}^{-1} \ dX \right] \text{Ad}_{\mathcal{g}_c}^{-1} \mathcal{g}_c + \left[ J(\bar{X})^T \mathcal{F}_p \right] = 0.
\end{align*}
\] (16)

Accounting for the members in square parenthesis, the generalized statics equation (16) can be written in the standard form as:

\[
\tau + N \text{Ad}_{\mathcal{g}_c}^{-1} \mathcal{g}_c + \mathcal{F} = 0,
\] (17)

where we have introduced the generalized internal and actuation load \( \tau(\bar{\xi}) = [\tau_1^T \ \tau_2^T \ \cdots \ \tau_N^T]^T \in \mathbb{R}^{6N} \), the generalized external concentrated load \( \mathcal{F}(\bar{\xi}) = [F_1^T \ F_2^T \ \cdots \ F_N^T]^T \in \mathbb{R}^{6N} \), and the gravitational matrix \( N(\bar{\xi}) \in \mathbb{R}^{6N \times 6} \).

### 3 TENDON AND FLUIDIC ACTUATION MODEL

In this section, a screw-based model of the tendon and fluidic actuation for soft manipulators is described. The tendon actuation for soft manipulator consists of built in cables running inside the soft body of the manipulator and following a certain path. One extreme of the cable is anchored at some point along the arm, while the other extreme is free to move behind the base of the manipulator and is used to pull the cable and actuate the system. Similarly, the fluidic actuation consists of built in reinforced chambers which run inside the soft body of the manipulator following different paths. Usually, both the extremes of the chamber lie inside the body and the pressurized fluid is brought from different channels. A depiction of the tendon and fluidic actuation is shown in Figure 2.

Consider the cable first, by the Newton’s third law, the distributed actuation exerted by the cable to the soft arm \( (\mathcal{\tau}_a) \) is opposite to the load experienced by the cable itself. Thus, from the steady-state equilibrium of the cable as extensible string ([13], [5]) evaluated in the (micro-)body frame we have:

\[
\mathcal{F}_i^* + \text{ad}_{\mathcal{g}_c}^* \mathcal{F}_i^* - \mathcal{F}_a = 0,
\] (18)

where \( \mathcal{F}_i^* \) is the cable internal load equal to \( \mathcal{F}_i^* = \text{Ad}_{\mathcal{g}_c}^* [0 \ 0 \ 0 \ T \ 0 \ 0]^T \), with \( T \in \mathbb{R}^+ \) the tension of the cable and \( g_c(X) \in SE(3) \) defining the position and orientation of the cable with respect to the midline of the arm (Figure 3-left). Equation (18) yields the screw-based model of the tendon actuation for soft manipulators:

\[
\mathcal{F}_a(X) = - \left( \mathcal{F}_a^* + \text{ad}_{\mathcal{g}_c}^* \mathcal{F}_a \right),
\] (19)

where we have defined the actuation field \( \mathcal{F}_a = -\mathcal{F}_i^* = \text{Ad}_{\mathcal{g}_c}^* [0 \ 0 \ 0 \ -T \ 0 \ 0]^T \), i.e. an extensible string experiencing a positive internal tension is pulling the surrounding.

Now, considering the fluidic chamber as an exten-
be analytically solved with an integration by parts with the ad-
load for the section $n$ holds for the case of fluidic actuation with
section $n$ of the torsional actuation load.

Introducing equation (19) in the equilibrium (16), each element $\tau_n$ of the generalized internal and actuation load becomes:

$$\tau_n = \sum_{j=n}^{N} \int_{L_{j-1}}^{L_j} S_n^T \left( \mathcal{F}_i - \mathcal{F}_n + \text{ad}^*_n (\mathcal{F}_i - \mathcal{F}_n) \right) dX,$$

where we have defined the $6 \times 6$ components of the Jacobian $J(X) = [S_1(X) \ S_2(X) \cdots S_N(X)]$. Notice that by definition $S_n(X) = 0_6$ for $X \leq L_{n-1}$.

Each of the integrals in the series above, except for the first one, can be solved analytically making use of the identity $\text{Ad}^*_n (\mathcal{F}^i + \text{ad}^*_n \mathcal{F}) = (\text{Ad}^*_n \mathcal{F})^i$, while the first one can be analytically solved with an integration by parts with the additional use of the identity $\text{Ad}^*_n (\mathcal{F})^i = \text{Ad}^*_n \mathcal{F}$. Applying these operations, we obtain the internal elastic and actuation load for the section $n$ as follows.

$$\tau_n = \sum_{j=n}^{N} \left( [S_n^T (\mathcal{F}_i - \mathcal{F}_n)]_{L_{j-1}}^{L_j} - \int_{L_{j-1}}^{L_j} \mathcal{F}_i - \mathcal{F}_n dX \right). \quad (20)$$

As observed before, the tendon actuation usually runs from the point of anchorage to the base of the manipulator while the fluidic actuation lies within one section of the arm. This brings to different boundary conditions for the two cases which are essential to calculate the sum in (20). For the cable-driven actuation case the boundary conditions are given by:

$$\mathcal{F}_i (L_n^+) = \mathcal{F}_i (L_{n+1}^-), \quad \mathcal{F}_i (L_n^-) = \mathcal{F}_i (L_{n+1}^+) + \mathcal{F}_{a_n},$$

$$\mathcal{F}_n (L_n^+) = \sum_{j=n+1}^{N} \mathcal{F}_j \mathcal{F}_n (L_n^-) = \sum_{j=n+1}^{N} \mathcal{F}_j + \mathcal{F}_{a_n}. \quad (21)$$

where the contribution of the cables attached at $L_n$ is indicated with $\mathcal{F}_{a_n}$ (Figure 2) and the constant internal load of the section $n$ with $\mathcal{F}_n$. As expected, crossing an anchoring edge $L_n$ causes a jump in both the internal elastic and actuation load, due to the concentrated load of the cables anchored at that position and the suddenly increase of the number of cable running through the section respectively. For the fluidic actuation case instead, the boundary conditions are:

$$\mathcal{F}_i (L_n^+) = \mathcal{F}_i (L_{n+1}^-) + \mathcal{F}_{a_n} - \mathcal{F}_{a_{n+1}},$$

$$\mathcal{F}_n (L_n^+) = \mathcal{F}_n (L_{n+1}^-) + \mathcal{F}_{a_n}, \quad (22)$$

where $\mathcal{F}_{a_n}$ here indicates the pushing load of the fluidic actuators at the section $n$ against the arm cross-section $L_n$ (Figure 2).

Substituting either equation (21) or (22) in (23), results in a brutal cancellation of the first term (the sum), which leads to the generalized internal and actuation load:

$$\tau_n = \int_{L_{n-1}}^{L_n} \mathcal{F}_a dX - (L_n - L_{n-1}) \mathcal{F}_{a_n}, \quad (23)$$

where we have exploited the constant strain assumption in $\mathcal{F}_{a_n}$.

In conclusion, the generalized internal and actuation load (23) is just the difference between the integral along the section of the inverse of the tendon/chamber internal load seen from the midline frame ($\mathcal{F}_n$) and a rescaling of the internal load of the arm ($\mathcal{F}_{a_n}$).

### 3.1 Spiral Path

The integral of the actuation field in (23) is determined by the actuation of the tendon/chamber (given by $T$ or $pA_c$) and more importantly by the relative position and orientation of the actuator with respect to the midline for each $X$ (given by $g_c$, Fig. 3-left). Imagine, for example, to have a spiral configuration of the actuator, centered on the midline, spinning anticlockwise with respect to the arm axis, at a distance $d$ from it, and inclined by an angle $\alpha$ with respect to the normal plane to the midline.

It can be shown that the actuation field at a generic $X$ is an anticlockwise rotation of the actuation at $X = 0$, thus

$$\mathcal{F}_a (X) = e^{(2\pi X / h) \text{Ad}^*_n \mathcal{F}_a (0)}, \quad (24)$$

where $h = d \tan(\alpha)$ is the pitch of the spiral configuration and $\xi_a = [1 \ 0 \ 0 \ 0 \ 0 \ 0]^T$. By introducing equation (24) in equation (23) leads to the integral of the exponential in (24) (from 0 to $L$ for simplicity) which is given by (8) by substituting $\text{ad}^*$ for $a_d$ and $\xi_a$ for $\xi_d$. In particular, for $L = h$ we obtain:

$$\tau_n = L (\text{diag}(1, 0, 0, 1, 0, 0)) \mathcal{F}_a (0) - \mathcal{F}_{a_n}, \quad (25)$$

i.e. the actuation load becomes a torsion followed by a contraction (tendon actuation) or elongation (fluidic actuation).

### 3.2 Constant Distribution Path

As it is clear from the example above, where we have lost the bending loads, the piece-wise constant strain method
introduces inevitably some approximation in the computation of the generalized actuation load corresponding to an averaging of the actuation along the section. Therefore, an interesting class of actuation fields is composed of scenarios which have a constant distributed actuation \( \mathcal{F}_a(X) \) given by (19). This class of actuation, satisfies:

\[
\mathcal{F}_a'' + \text{ad}_{\xi_a} \mathcal{F}_a' = 0,
\]

which is obtained by taking the derivative of (19) as seen from an observer sited on the (micro-)body frame.

Solving equation (25), it can be shown that for the actuation field \( \mathcal{F}_a(X) \) to satisfy equation (25) for every possible value of the strain \( \xi_a \), it has to be constant along \( X \) or, in other words, the tendon/chamber paths have to be parallel to the midline of the arm and at a constant position with respect to it (Fig. 3-right). In this condition, the generalized internal actuation and actuation load (23) becomes (for the tip-to-base tendon actuation and the section-wise fluidic actuation respectively):

\[
\tau_a = (L_m - L_{n-1}) \left( \sum_{i=1}^{n} \sum_{j=1}^{m_i} \mathcal{F}_{a1} - \mathcal{F}_a \right), \quad (26)
\]

\[
\tau_a = (L_m - L_{n-1}) \left( \sum_{i=1}^{m} \mathcal{F}_{a1} - \mathcal{F}_a \right), \quad (27)
\]

where \( \mathcal{F}_{a1} \) (i \( \in \{1, 2, \ldots, m_i\} \)) is the constant contribution of one actuator and \( m_a \) is the number of actuators of the section \( n \).

4 ACTIVE DOFs SUBSPACE

One of the main features of soft robotics is the ability to adapt to the environment due to its extreme hyper-redundancy. As a matter of fact, a soft arm has virtually infinite DOFs, while applying the pie-wise constant strain method we have reduced the number of DOFs to the finite \( 6N \). However, even in the reduced case, not all the DOFs are actuated, thus for control and operational purpose it is important to model the active DOFs subspace. The purpose of this section is to develop an active DOFs subspace model, specially for the case of constant distribution actuation.

4.1 Actuation and Active DOFs Subspaces Duality

The relation between actuation load and the active DOFs is given by the steady-state equilibrium (17) evaluated in absence of external load, which yields: \( \mathcal{T} = 0 \), and for one section labeled \( n \): \( \tau_n = 0 \). In the general case (equation (23)), defining the basis for the normalized (with \( L_m - L_{n-1} \)) integral in (23) as the actuation basis \( B_{a1} \in \mathbb{R}^{6 \times k} \) and the corresponding basis for the active DOFs as \( B_{a} \in \mathbb{R}^{6 \times k} \), we obtain:

\[
B_{a1} \lambda_a = \Sigma B_{a1} \xi_a,
\]

where we have defined the vector of actuation loads \( \lambda_a \in \mathbb{R}^k \) and active DOFs \( \xi_a \in \mathbb{R}^k \) and we have used the constitutive equation (13) with \( B_{a1} \xi_a = \xi_a - \xi_0 \), \( \xi_a \) being the active part of the strain \( \xi_a \). In order to establish a relation between the actuation basis and the active DOFs basis, we chose an arbitrary scaling between \( \lambda_a \) and \( \xi_a \) as follows: the \( j \)-th element of \( \xi_a \) is equal to the \( j \)-th element of \( \lambda_a \) multiplied by \( \|k_j\| / \|E_{\theta} k_j\| \) where \( k_j \) is the rotational part of the \( j \)-th column of \( B_{a1} \) and \( E_{\theta} \) is the rotational part of the screw stiffness matrix \( \Sigma \). Alternatively, if \( k_j = 0 \) we multiply by \( \|q_j\| /\|E\_\theta q_j\| \) where \( q_j \) is the linear part of the \( j \)-th column of \( B_{a1} \) and \( E \) is the linear part of the screw stiffness matrix \( \Sigma \). Let us call the matrix built in this way as \( \xi \in \mathbb{R}^{k \times 6} \), which allows us to normalize the active DOFs basis: \( \xi_a = \Lambda \xi_a \). Then, from equation (28), we obtain the following relation between the two basis:

\[
B_{a1} v = \Sigma^{-1} B_{a1} v = \Lambda^{-1} v .
\]

On the other hand, the passive load basis \( B_{p1} \) and the passive DOFs basis \( B_{p} \) are defined as the \( \mathbb{R}^{6 \times 6-k} \) matrix with linearly independent columns which are reciprocal to respectively \( B_{a1} \) and \( B_{a} \). In other words, they satisfy respectively: \( B_{p1}^T B_{p1} = 0 \) and \( B_{a1}^T B_{p1} = 0 \). Let us call the vector of passive loads \( \lambda_p \in \mathbb{R}^{6-k} \) and the vector of passive DOFs \( \xi_p \in \mathbb{R}^{6-k} \), then, we can express a generic external load \( F_n \) and a generic configuration strain \( \xi_n \) as: \( F_n = B_{a1}^T \lambda_a \) and \( \xi_n = (B_{a1} \xi_a + \xi_0) + B_{p1} \xi_p \).

This allows us to state a first necessary condition for force-closure: a soft manipulator section can resist an applied force only if the letter has no components on the passive DOFs subspace, i.e. only if \( B_{p1}^T F_n = 0 \) holds.

4.1.1 Constant Distribution Actuation Dual

Let us now apply the general development above to the specific case of constant distribution actuation found out in section 3.2 and expressed by equations (26) and (27). Consider the configuration depicted in Figure 3-(right) for one tendon and one fluidic actuator, the actuation load (\( B_{a1}^T \lambda_a \)) and the corresponding active DOFs through equation (29) \( (B_{a1} \xi_a + \xi_0 = \xi_n) \) are as follows:

\[
\mathcal{F}_{a1} \Rightarrow \begin{bmatrix} 0 & d & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} T \quad \xi_n = [0 \ d \ 0 \ 0 \ 0 \ \xi_0]^T \frac{T}{E J} \quad \text{if} \ d \neq 0
\]

\[
\mathcal{F}_{a1} \Rightarrow \begin{bmatrix} 0 & d & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} T \quad \xi_n = [0 \ d \ 0 \ 0 \ 0 \ \xi_0]^T \frac{T}{E A} \quad \text{if} \ d = 0
\]

\[
\mathcal{F}_{a1} \Rightarrow \begin{bmatrix} 0 & d & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{pA_c}{E J} \quad \xi_n = [0 \ d \ 0 \ 0 \ 0 \ \xi_0]^T \frac{T}{E A} \quad \text{if} \ d \neq 0
\]

\[
\mathcal{F}_{a1} \Rightarrow \begin{bmatrix} 0 & d & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{pA_c}{E A} \quad \xi_n = [0 \ d \ 0 \ 0 \ 0 \ \xi_0]^T \frac{T}{E A} \quad \text{if} \ d = 0
\]

respectively for the tendon and fluidic actuation, where \( d \) is the constant distance between the actuator and the arm midline.

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4.2 Screw System of Constant Distribution Actuation

A well known screw theory result is that a twist ∈ se(3) can be associated with a geometric screw. As described in [19], a screw consists of an axis ̂a, a pitch h and a magnitude m and a screw motion represents rotation by an amount m about the axis ̂a followed by translation by an amount hm parallel to the axis ̂a. It can be shown that a screw motion corresponds to a motion along a constant twist by an amount equal to the magnitude of the screw, which allows us to associate a screw with every twist. In the context of this paper, the constant strain of a section ̂ξ is defined in the (micro-)body frame, nevertheless, since it is associated with all the possible configuration of a constant distribution actuation.

In the following, in order to analyze the active DOFs subspace for constant distribution actuation, we will calculate the parameters of the screws given by the twist vectors ̂ξn of (30), which will allow us to find the screw system associated with all the possible configuration of a constant distribution actuation.

4.2.1 Screw Geometry

First of all, let us recall that, the strain configuration ̂ξn is defined in the (micro-)body frame, nevertheless, since it is itself the infinitesimal generator of the maps gα(X), it can be equivalently imagined to be defined at the base of the section X = Ln−1 through the equivalence: Adgα(X) ̂ξα = ̂ξα, where we have used the derivative of equation (8) and the anticommutativity of the adjoint map. For this reason, the resulting screw parameters are obtained with respect to the frame at the base X = Ln−1.

The screw parameters relative to ̂ξn are obtained by adapting the formulas normally used for time-twist [19], which become:

\[
\begin{align*}
h_{an} &= \begin{cases} \frac{\kappa_{an} \cdot q_{an}}{L_{n-1} - h} & \text{for } d \neq 0 \\ \infty & \text{for } d = 0 \end{cases} \\
\hat{a}_{an} &= \begin{cases} \frac{\kappa_{an} \cdot q_{an} + \delta k_{an}}{L_{n-1} - h} & \text{for } d \neq 0 \\ \frac{\delta q_{an}}{L_{n-1} - h} & \text{for } d = 0 \end{cases} \\
m_{an} &= \begin{cases} \frac{(X - L_{n-1}) \theta_{an}}{\sqrt{q_{an}^T q_{an}}} & \text{for } d \neq 0 \\ \frac{(X - L_{n-1}) \sqrt{q_{an}^T q_{an}}} & \text{for } d = 0 \end{cases}
\end{align*}
\]

(31)

where, by convention, a screw motion with h = ∞ represents a pure translation. For completeness, let us shown how, vice versa, the strain ̂ξn may be obtained from the screw parameters. For the cases of pure translation and full rigid body motion we have respectively:

\[
\begin{align*}
\tilde{\xi}_{an} &= m_{an} - \hat{a}_{an} + \delta k_{an} \\
\hat{\xi}_{an} &= \frac{\kappa_{an} \cdot q_{an}}{L_{n-1} - h} - \hat{a}_{an} + h_{an} k_{an}.
\end{align*}
\]

(32)

where qαn and kαn are unit vector in this case and p represents any point of the axis ̂aα expressed in the base frame of section n.

In equation (31), whenever applicable, the plus sign refers to the fluidic actuation with ̂ξα = pAℓ/EJ, while the minus sign refers to the tendon actuation with ̂ξα = T/EJ and for simplicity we have taken Ln−1 = 0. Notice that, due to the constrain of material non-penetration, the x-component (tangent to the midline) of ̂ξα is always positive and consequently we have 1/̂ξα ± J/A > 0 and 1 ± ̂ξα > 0. As depicted in Figure 4, we end up with either pure rotation screws located on the left side of the z-axis of the base (for d ≠ 0) or pure translation screws along the x-axis of the base (for d = 0). It is worth noticing here that this configuration represents exactly the circular geometry of the PCC model, a special case of screw geometry with pitch either equal to zero or infinity. Accordingly, the radius of curvature, bending plane and circular angle of the PCC model are given respectively by kαn/θ2α, κn and mα.

The screws obtained in (31) can be moved horizontally by increasing d or ̂ξα, which leads to the approach the arm, and vice versa by decreasing them, which leads to move away from the arm. Furthermore, a rotation around x of the position of the actuator causes the screws to rotate in the same way. Using those two movements, the entire plane containing the base can be covered with pure rotation screws including the one at infinity (pure translation along x), pointing anticlockwise due to the non-penetration constraint (Fig. 4). This plane is known as the characteristic plane of the screw system spanned by \{ ̂ξα, ̂ξα, ̂ξ α \}, which are the pure unitary rotation around y, the pure unitary rotation around z and the pure unitary translation along x respectively. Thus we obtain:

\[
\tilde{\xi}_{an} \in \text{span} \{ \tilde{\xi}_\alpha, \tilde{\xi}_\alpha, \tilde{\xi}_\alpha \} := m \in se(3),
\]

(33)

for every possible ̂ξn, which identifies the active DOFs subspace generated by a constant distribution actuation.

To plan and follow configuration paths for a chain of sections composing a soft manipulator, it is important to know the geometric properties of the active DOFs subspace.
5.1 Experiments

APPLICATION TO STIFF-FLOP

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5.1 Experiments

Two sets of tests were made. One series considered the module in the vertical orientation, such that the axis of the manipulator is aligned with the line of action of gravity. The other set was in the horizontal regime with the gravity acting as an out of plane actuation force. In order to account for the out of plane bending, a load of 8.5 grams was attached to the end of the manipulator. Tests were carried out to verify effects in the high pressure regime as well as the low pressure regime.

A rigid frame has been used to maintain the module base in a fixed position for both the horizontal and the vertical orientation. The tracking system has been positioned above the
setup, so that the probe remains within its workspace also during tip motion. This allowed the measurements of the position and orientation of the tip while the chambers are activated in different combinations. The activation patterns have been selected considering the basic module capabilities and symmetries. These measurements are with respect to a stationary frame of the tracking system, and thus have to be transformed by the use of a transformation matrix to be relative to the fixed end of the robot. In this manner the development of the $g(L_0)$ (with $n$ equal to 1) matrix can be facilitated. The Young’s modulus and Poisson’s ratio used for the model are 89.7 kPa and 0.5 respectively. In terms of the geometrical dimensions, a length of 44 mm, lumen radius of 2.25 mm and a chamber radius of 1.4 mm is used. The considered length is only of the soft silicone body. The radius of the module is considered to be 7.5 mm. The chambers are considered to be 5.2 mm from the center of the module and have an angle of 15° degrees between them.

5.2 Results and Discussion

For each test, the $g(L) = g_1(L)$ map (where $L$ is the length of the module) is extrapolated from the measurements, and, by means of equation (5), the strains $\xi$ of the module is obtained. This allows us to calculate the position and orientation of each point of the arm $g(X)$ through (3) as well as the integral map $AD_{\xi}(X)$ with (8) and the geometric Jacobian $J(X)$ as defined in (10). At this point, the external and elastic loads are readily obtained, and the steady state equilibrium (17) can be solved for the actuation load $\sum_{i=1}^{n} F_{\tau i}$ ($i \in \{1,2,3\}$) where $\tau$ is given by (27). Finally, the contribution of each of the three chamber is derived since the actuation load belong to the three dimensional $m^*$ and each contribution is linearly independent.

Figure 6 shows the chamber pressures predicted by the model against the actual pressures supplied to the chambers. As can be seen, the model predicts the pressures in all regimes (i.e. elongation, vertical and horizontal bending) with reasonable accuracy. The error is calculated as the difference between the actual and predicted pressure for each chamber normalized over the highest pressure for that test. There are two basic sources of errors. Firstly they are due to the variation of the parameters of the module. The fabrication of soft robots in general is not exactly reproducible, and can bring about changes not only in length and diameters, but also the Young’s modulus. Some other added factors are the uncertainties in the sensor placement, the exact values of the pressures inside the chambers and the location of load application point. Since these factors can have significant sensitivities, the parameter values were identified for the model separately. Secondly, the assumption of constant strain itself is not completely correct. However as can be seen, the final result is within a respectable accuracy range of 10%.

6 CONCLUSIONS

In conclusion, a screw-based formulation for the soft manipulators kinematics, differential kinematics and statics based on the Piecewise-Constant Strain assumption has been presented. Results include the development of the product of exponentials formula and the geometric Jacobian for soft robotics kinematics as well as a steady-state multisection model for soft robotics.

A screw-based formulation of the actuation load model exerted by embedded tendons and fluidic chambers has also been presented. This led to the identification of the screw system generated by the all possible configurations of a constant distribution actuation, whose properties have also been detailed. Reciprocity arguments allowed us to clearly dif-
differentiate between active and passive DOFs subspaces providing a guideline for the assessment of force-closure and manipulable conditions.

Finally, the model has been validated through experiments with the soft manipulator for minimally-invasive surgery STIFF-FLOP, achieving good results. During this process, the steady-state model has been inverted by mean of the logarithmic function and the actuation pressure deduced.

In the authors’ opinion, the present screw-based formulation, highlights the real geometric structure of soft manipulators and provides new design and control insights with respect to the traditional Piecewise-Constant Curvature approach, hence representing a milestone of its own kind.

References


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