UNIFIED KINEMATICS ANALYSIS AND ANALYTIC SINGULARITY-FREE WORKSPACE OF A METAMORPHIC PARALLEL MECHANISM WITH CONTROLLABLE ROTATION CENTER

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ABSTRACT
This paper presents a metamorphic parallel mechanism with controllable rotation center in its pure rotation topology. Based on reconfiguration of a reconfigurable Hooke (rT) joint, the rotational center of the mechanism can be altered along the central line perpendicular to the base plane. A unified Dixon resultant based method is proposed to solve the forward kinematics analytically by covering all configurations with variable rotation centers while the rotation motion is expressed using Cayley formula. Then singularity loci are derived and represented in a new coordinate system with the three Rodrigues-Hamilton parameters assigned in three perpendicular directions. Limb-actuation singularity loci are also obtained from row vectors of the Jacobian matrix. By using Cayley formula, analytical workspace boundaries are expressed by including the mechanism structure parameters and input actuation limits. Finally, singularity-free workspace of configurations with variable rotation centers is demonstrated in the proposed coordinate system.

INTRODUCTION

Based on their multi-chain structures, parallel mechanisms [1] show great advantages in industrial, domestic and research applications requiring high stiffness, low inertia and good accuracy. Keeping those advantages of traditional parallel mechanisms, metamorphic parallel mechanisms [2] represent a class of parallel mechanisms that are reconfigurable to adapt to different application requirements, like machining simple to complex mechanical parts in industry, rehabilitation for different joints in human therapy. Reconfiguration of the parallel robots helps increase precision, simplify control, save energy and reduce risk by removing extra degrees of freedom.

Based on the principle of metamorphosis and mobility change, a patented reconfigurable Hooke joint [2] was introduced and a class of metamorphic parallel mechanisms was presented while a general synthesis method was given using screw theory [3]. This contributed to provide a new way in reconfiguring parallel mechanisms by changing joint axis constraint while the other way relies on link coincidence and locking [4]. Related work of metamorphic mechanisms started from 1990s with work on reconfigurable packaging [4] and kinematotropic linkages [5]. Later and recent research includes ortho-planar metamorphic mechanisms [6], metamorphic descriptions [7,8], kinematotropic parallel mechanisms [9],
metamorphic parallel mechanisms [3,10] and parallel mechanisms with multiple operation modes [11].

The mechanism investigated in this paper has three topologies with pure translational, pure rotational and 3T1R degrees of freedom. In addition to the mobility change, the rotation center of the pure-rotation topology can be also reconfigured along the line perpendicular to the base plane. A similar property was discussed with a P5 limb [12]. This is very novel comparing with other spherical parallel mechanisms [13-16] of which the rotation centers are normally fixed in the design. Much work has been carried out on spherical parallel mechanisms due to their wide applications including robot wrist [13, 14, 15] and rehabilitation [16]. Among the research topics on spherical parallel mechanisms, kinematics and workspace analysis attracted much attention. Various novel kinematics models [17-19] were proposed and solved analytically to get the eight forward kinematics solutions. In the workspace representation, tilt-and-torsion (T&T) angle method showed great advantages on both workspace illustration and singularity analysis [20]. Rodrigues parameters were used in [15] to study singularities but not for singularity-free workspace presentation. This paper introduces the Cayley formula [21] to represent the rotation, which demonstrates singularity loci with clear physical meaning, and shows a new approach to obtain analytical expressions for workspace boundaries. In addition to these, the unified model also presents clear singularity-free workspace.

The paper is arranged in the following structure. Section 1 introduces the geometric structure of the metamorphic parallel mechanism, its controllable rotation center and kinematics analysis. The Jacobian matrix is obtained in section 2 which presents singularity loci in a new coordinate system. Section 3 shows analytic expressions of workspace boundaries with example singularity-free workspace illustrations for variable rotation centers. Conclusions are made in section 4.

1. UNIFIED KINEMATICS ANALYSIS OF THE 3-(rT)P(rT) METAMORPHIC PARALLEL MECHANISM WITH CONTROLLABLE ROTATION CENTER

1.1 Geometric structure and reconfiguration of The 3-(rT)P(rT) metamorphic parallel mechanism

The 3-(rT)P(rT) metamorphic parallel mechanism consists of a base, a platform and three (rT)P(rT) limbs as in Fig. 1. Each limb has a prismatic joint with two reconfigurable rT joint [2] at the two ends and the ring parts of the rT joints are connected to the base and platform respectively. For convenience, the rT joint connected to the base is called base rT joint and the other called platform rT joint. The three limbs support the platform symmetrically around a reference circle of radius rA and connect to the base around a circle of radius rB. All the radial axes of the rT joints intersect at point O as in Fig. 1 in which the three limbs are numbered as limb 1, limb 2 and limb 3. Let B_i in the base denote the centre of the base rT joint and A_i denote the center of the platform rT joint in the ith (i = 1, 2, 3) limb and A_0 the geometric center of the platform. Locate a global coordinate frame Ox'y'z at the rotation center O with z-axis perpendicular to the base and y-axis in the same plane with line OB_1.

Screw based analysis [2] for the configuration in Fig. 1 showed that each limb provides a constraint force passing through point O and perpendicular to the plane OA_iB_i. Thus there are three constraint forces intersecting at point O and three translational DOFs of the moving platform are constrained resulting in that the 3-(rT)P(rT) parallel mechanism has pure rotational DOFs.

Based on the configuration in Fig.1, when altering the radial axes of the two rT joints in each limb to be parallel to each other, the mechanism will have pure translational DOFs. Specially, when the two parallel radial axes are perpendicular to the base plane, the mechanism becomes a 4-DOF configuration of 3-(rT)P(rT) with three translations and one rotation about z-axis [2]. In this paper, the focus is on the pure rotational topology.

In each limb as in Fig.1, the radial axes of the base rT joint forms an angle α_a with z-axis and hence called base-rT-joint angle, the radial axis of the platform-rT-joint forms an angle α_p with z'-axis and hence called platform-rT-joint angle. Thus the base radius r_a and base-rT-joint angle α_a describe the geometric structure of the base while the platform radius r_p and platform-rT-joint angle α_p show the geometric structure of the platform. As the radial axis of the rT joint can be rotated about the bracket axis by any angle, directions of the radial axes v_i and u_i in each limb can be altered freely, hence the intersecting point O can be moved along z-axis as in Fig. 2.

Assuming that the platform is at the initial configuration, the intersecting point O is changed from O to O' by rotating the radial axes of the rT joints, base-rT-joint angle α_a and platform-rT-joint angle α_p are changed to α_a' and α_p' respectively, the constraint equation among these parameters can be obtained as:

$$\pm \Delta h = r_b \cot \alpha_b - r_a \cot \alpha_a' = r_b \cot \alpha_a - r_a \cot \alpha_a'$$

(1)
where Δh is the distance between O and O'. When the change from O to O' is to the positive direction of z-axis, the sign before Δh is ‘+’, otherwise it is ‘−’.

\[ s_i = \left[ \cos(3\pi / 2 + 2\pi(i-1)/3), \sin(3\pi / 2 + 2\pi(i-1)/3), -\cot\alpha_a \right]^T, \]
\[ a'_i = r_b s_{i+5}, \quad (i = 1, 2, 3) \]

where 0 < a_a < \pi.

Let R be the 3×3 rotational matrix denoting the orientation of the moving coordinate frame with respect to the global coordinate frame, the closed-loop equation of each limb can be expressed in the global coordinate frame as

\[ (Ra'_i - b_i)^\top (Ra'_i - b_i) = d_i^2 \quad (i = 1, 2, 3) \] (4)

Expanding (4) gives:

\[ 2(Ra'_i)^\top b_i = (r_b / \sin\alpha_a)^2 + (r_b / \sin\alpha_a)^2 - d_i^2 \]
\[ = 2r_b^2 \cos(\phi) / (\sin^2\alpha_a \sin^2\alpha_b) \quad (i = 1, 2, 3) \] (5)

which can be described as

\[ (R v_i)^\top u_i = \cos(\phi) \quad (i = 1, 2, 3) \] (6)

where v_i and u_i denote the unit vectors along OA_i and OB_i in the platform coordinate system and the base coordinate system respectively, and \( \phi \) is the angle between axes v_i and u_i, Fig. 1.

When R is known, the inverse kinematics of the 3-(rT)P(rT) parallel mechanism is to get the input \( d_i \) which can be obtained directly from (4).

1.3 Forward kinematics

The forward kinematics analysis is to solve the orientation by giving the three limb length inputs. When the rotation center is changed, the platform has a pure rotation motion about the new rotation center. The following kinematics method can be used for all cases and is a unified method covering all the configurations. In the following, Cayley formula will be introduced to represent the platform orientation and Dixon’s resultant will be used to obtain the analytical solution of the forward kinematics analysis.

In the three-dimensional space, a rotation matrix R between two coordinate systems can be given by Cayley formula [20]:

\[ R = \Delta^{-1} \left[ 1 + c_1 c_2 - c_2^2 c_3 - c_2 c_1 c_3 \quad 2(c_2 c_1 - c_3) \quad 2(c_1 c_3 + c_2) \\
2(c_2 c_1 + c_3) \quad 1 - c_1^2 + c_2^2 - c_3^2 \quad 2(c_1 c_3 - c_2) \\
2(c_1 c_3 - c_2) \quad 2(c_2 c_1 + c_3) \quad 1 - c_1^2 - c_2^2 + c_3^2 \right] \] (7)

where \( \Delta = 1 + c_1^2 + c_2^2 + c_3^2 \), \( c_1 \), \( c_2 \) and \( c_3 \) are the Rodrigues-Hamilton parameters [15].

Substituting (7) into (6), simplifying and taking the numerators, gives:

\[ f_i(1, c_1, c_2, c_1, c_1, c_2, c_1, c_1, c_2, c_1, c_2, c_1, c_2, c_1) = 0 \quad (i = 1, 2, 3) \] (8)
where \( f(t) \) is a function of the unknown power products in the bracket, with real constant coefficients depending on the input and mechanism dimension parameters only.

Rewrite (8) as the function of \( c_1, c_2 \) by putting the products of power in \( c_3 \) into the coefficients:

\[
Q_e(c_1, c_2) = \sum_{i,j \neq 0} G_{e-y} c_i c_j' = 0 \quad (e = 0, 1, 2) \quad (9)
\]

where \( G_{e-y} \) are functions of \( g_{e-jk} \) and power products of \( c_3 \).

Construct the following matrix

\[
\Delta(c_1, c_2, t_1, t_2) = \begin{bmatrix}
Q_0(c_1, c_2) & Q_1(c_1, c_2) & Q_2(c_1, c_2) \\
Q_0(t_1, t_2) & Q_1(t_1, t_2) & Q_2(t_1, t_2)
\end{bmatrix}
\quad (10)
\]

where \( t_1, t_2 \) are intermediate parameters only. By developing the above equation, \( \Delta(c_1, c_2, t_1, t_2) \) is a polynomial of order 2 in \( c_1, 3 \) in \( c_2, 3 \) in \( t_1 \), 2 in \( t_2 \). Dixon observed that \( \Delta \) vanishes when \( Q_0(c_1, c_2), Q_1(c_1, c_2) \) and \( Q_2(c_1, c_2) \) have common zeros no matter what \( t_1, t_2 \) are. The coefficients of each power product \( t_i t_j^i \) \((i=0,1,2; j=0,1)\) of \( \delta \) have common zeros which are also the common zeros of equations \( Q_0, Q_1, Q_2 \). This gives five equations in power product of \( c_1 \) and \( c_2 \), whereas the number of the power product \( c_i'c_j' \) \((i=0,1; j=0,1,2)\) is also five. Therefore, the coefficients of each power product \( c_i'c_j' \) in these five equations form a 5×5 matrix \( D \). All the above algorithm can be expressed as:

\[
\delta(c_1, c_2, t_1, t_2) = \frac{\Delta(c_1, c_2, t_1, t_2)}{(c_i-t_i)(c_j-t_j)} = 0
\quad (11)
\]

is a polynomial of order 1 in \( c_i, 2 \) in \( c_j, 2 \) in \( t_1, 1 \) in \( t_2 \). From (11), the determinant of \( D \) equals 0. Thus an equation in \( c_3 \) can be obtained:

\[
|D| = \sum_{i=0}^{5} h_i c_i' = 0
\quad (13)
\]

where \( h_i \) are real constants depending on input data only.

This implies that an univariate equation in \( c_3 \) of order 8 is obtained. Solving (13), all the solutions for \( c_3 \) can be obtained. Then substitute \( c_3 \) into the following equation:

\[
D C' = 0
\quad (14)
\]

According to the Cramer's rule, the solutions of \( c_1, c_2 \) can be computed from the above linear system. Substituting all the solutions of \( c_1, c_2 \) and \( c_3 \) to (7), orientation \( R \) can be obtained.

### 2. SINGULARITY LOCI

#### 2.1 Jacobian matrix and 3D singularity loci

Taking the derivative of (2), there is

\[
\begin{bmatrix}
(b_1 \times (R a_1'))^T \\
(b_2 \times (R a_2'))^T \\
(b_3 \times (R a_3'))^T
\end{bmatrix} \omega = \begin{bmatrix}
J_1 \\
J_2 \\
J_3
\end{bmatrix} \omega = J \omega = \begin{bmatrix}
d_1 \\
d_2 \\
d_3
\end{bmatrix}
\quad (15)
\]

where \( \omega \) is the platform orientation velocity, \( \dot{a} \) is the input velocity of limb \( i \), and \( J=(b_i \times (Ra_i'))^T \) is the row vector of the Jacobian matrix \( J \). Hence Type 2 singularities result when the determinant of \( J \) equals to zero. Based on the rotation matrix \( R \) in (7) and the symmetrical structure, the determinant of \( J \) is given by:

\[
|J| = f_a (c_1, c_2, c_2, c_2, c_2, c_1, c_1, c_2, c_1, c_2, c_1)
\quad (16)
\]

From (16), the determinant of the Jacobian matrix is a quartic polynomial of the three rotation elements \( (c_1, c_2, c_3) \) with coefficients consisting of structure parameters. For a given structure of the 3-(rT)P(rT) parallel mechanism, the rotation elements \( (c_1, c_2, c_3) \) can be used to represent the singularity configuration of the platform. By equating (16) to zero, all the singular points can be found as examples in Fig. 3.

![SINGULARITY LOCI](image-url)
In Fig. 3, singularity loci associated with different rotation centers are illustrated. In this example, the parameters used are \( r_s=1, r_l=2, \alpha_s=\pi/3 \) and \( \alpha_s=2\pi/9 \). By giving the desired platform angle \( \alpha'_u \) representing new rotation center, corresponding base angle \( \alpha'_b \) can be calculated from (1) and the determinant polynomial of the Jacobian matrix in (16) is renewed. In general, it can be seen that the singularity loci are symmetrical around the \( c_1 \) axis which represents the symmetrical locations of the three limbs between the platform and the base. A special case is the example in Fig. 3 (c) in which \( \alpha'_u=\alpha'_b \approx 0.907 \) of which the platform and base have the similar shape but different sizes. When \( \alpha'_b \) is further from this configuration, the singularity loci are more curved as seen in Fig. 3(a) when \( \alpha'_u=\pi/12 \) while the singularity loci are similar for the cases in Fig. 3(b) with \( \pi/4 \) and Fig. 3(d) with \( 7\pi/18 \). For the special case in Fig. 3(c), (16) becomes a polynomial of order 3 in \( c_1 \) and order 2 in \( c_2 \) with a factor \( c_3 \):

\[
|J| = f_j (c'_1, c'_1, c'_2, c'_2, c'_2) \tag{17}
\]

In (17), \( c_3 \) is a factor of the determinant, and hence the plane \( c_3=0 \) is a part of the singularity loci, and the other parts of the singularity loci depend only on \( c_1 \) and \( c_2 \). This can be seen from Fig. 3(d) in which the singularity loci consists of the \( c_3=0 \) plane, and three other scattered parts and a central part on surfaces perpendicular to the \( c_3=0 \) plane. The singularity loci surface has clear symmetry on the \( c_1; c_2 \) plane as a result of the symmetrical limb arrangement and is also symmetrical with respect to the \( c_2=0 \) plane, due to the elements of \( c_2 \) in the Jacobian matrix determinant being quadratic only.

### 2.2 Limb actuation singularity

Limb actuation singularity [20] defines configurations where the limb cannot be actuated even when the other limb actuation joints are released. In the 3-(rT)P(rT) metamorphic parallel mechanism, it happens when a limb passes through the rotation center \( O \). The following analysis shows a new method to calculate the limb actuation singularity loci.

Based on the Jacobian matrix in (15), limb actuation singularities can be found by making the row vector \( J_i = (J_{i1}, J_{i2}, J_{i3}) = 0 \). In general, the components \( J_{ij} \) are quadratic polynomials of the rotation elements \( c_1, c_2, c_3 \) and \( J_i = (J_{i1}, J_{i2}, J_{i3}) = 0 \) gives three curved surfaces intersecting at two lines \( l_{11} \) and \( l_{12} \) which are perpendicular to each other. The physical meaning of lines \( l_{11} \) and \( l_{12} \) is when the platform rotates to any configuration where \( OA_1 \) is collinear with \( OB_1 \), in the same direction with \( Rv_i = u_i \), the point \( (c_1, c_2, c_3) \) is on the line \( l_{11} \). When \( OA_1 \) is collinear with \( OB_1 \), in the opposite direction, with \( Rv_i = -u_i \), the point \( (c_1, c_2, c_3) \) is on the line \( l_{12} \). Since the mechanism is symmetrical and the limbs have the same structure, the limb singularities for limb 2 and limb 3 are the same and each of them has two intersection lines \( (l_{21}, l_{22}, l_{31}, \text{and} \ l_{32}) \) from three curved surfaces based on \( J_i = (J_{i1}, J_{i2}, J_{i3}) = 0 \).

Their location relative to the mechanism singularity loci is illustrated by combining the limb singularity lines with the singularity loci in Fig. 4 in which the rotation center is not changed in the example in Fig. 3 with \( \alpha'_u=\alpha'_b=2\pi/9 \). In Fig. 4, lines \( l_{11}, l_{21}, \text{and} \ l_{31} \) are nearly parallel to \( c_3=0 \) plane while the lines \( l_{12}, l_{22} \) and \( l_{32} \) are nearly perpendicular to the \( c_3=0 \) plane. When coming to the special case with \( \alpha'_u=\alpha'_b \), the former three lines are in the \( c_3=0 \) plane and the latter three are perpendicular to it.

![Figure 4. Limb Actuation Singularity Loci in the Mechanism Singularity Loci (\( \alpha'_u=2\pi/9 \))](http://proceedings.asmedigitalcollection.asme.org/)

### 3. Analytical Singularity-Free Workspace

#### 3.1 Analytical Description of the Workspace Boundaries

In section 2, the rotation elements \( c_1, c_2 \) and \( c_3 \) are used to illustrate the singularity loci by using a 3D coordinate system, \( O-c_1c_2c_3 \). This can be extended to represent the rotation workspace using the same coordinate system with \( c_1, c_2 \) and \( c_3 \) in three perpendicular directions. According to the physical meaning of the Rodrigues-Hamilton parameters, a point \( C(c_1, c_2, c_3) = \tan(0/2)^*(kx, ky, kz) \) corresponds to a platform rotation by angle \( \theta \) about an axis \( k(kx, ky, kz) \) in the mechanism base coordinate system. Thus, the workspace coordinate system, \( O-c_1c_2c_3 \), is parallel with the mechanism base coordinate system, \( O-axis \) with coincident centers \( O \). This property shows that the \( O-c_1c_2c_3 \) coordinate system has intuitive physical meaning in representing the rotation workspace.

In the 3-(rT)P(rT) metamorphic parallel mechanism, each limb length has two limits (lower and upper) which constrain the actuation range and determines the rotation workspace of the platform. Based on (5), the limb length limits will result in lower and upper limits of angle \( \phi_i \) between the platform and base vectors \( v_i \) and \( u_i \) in each limb. Thus, the platform rotation workspace boundaries can be expressed by the two limits \( \phi_{i\text{max/min}} \) by calculating the triangle relation, using (6) as:

\[
(Rv_i)^T u_i = \cos(\phi_{i\text{max/min}}) \quad (i = 1, 2, 3) \tag{18}
\]

For limb 1, it can be expanded as:
\[ k_0 + k_c^1 + k_c^2 + k_c^3 + k_i^1c_1 + k_i^2c_2 + k_i^3c_3 - \Delta \cos(\phi_{\text{max/min}}) = 0 \]  

(19)

where \( \Delta = 1 + c_i^1 + c_i^2 + c_i^3 \) as in (7), \( k_i \) are coefficients depending on the mechanism structure parameters only.

Equation (19) shows the advantage of the \((c_1, c_2, c_3)\) coordinate system, since the coefficients of \( c_1 \) and its square consist of mechanism structure parameters only, \( c_1 \) can be easily expressed using \( c_2 \) and \( c_3 \) with coefficients including the angle limits. This makes it possible to have analytical expressions to describe the 3D rotation workspace using the three rotation elements. From (19), \( c_1 \) can be solved as:

\[
c_1 = \frac{-k_i \pm \sqrt{k_i^2 - 4k_k c_k}}{2k_k}
\]

(20)

where

\[
k_k = k_c - c \cos(\phi_{\text{min}}) \sin(\alpha),
\]

\[
k_c = k_0 - c \cos(\phi_{\text{min}}) \sin(\alpha) + (k - c \cos(\phi_{\text{min}}) \sin(\alpha))c^2 +
\]

\[
k_4c \cos(\alpha) \pm (k - c \cos(\phi_{\text{min}}) \sin(\alpha))c^2
\]

Thus, analytical workspace boundary from limb 1 can be given in the \(O-c_1c_2c_3\) coordinate system using (20) with input limits \(\phi_{\text{min}}\text{--}\text{max}\). Since the limb arrangement in the 3-(rT) metamorphic parallel mechanism is symmetrical, workspace boundaries corresponding to each of the three legs have a similar shape, but rotated \(2\pi/3\) about the \(c_3\) axis. Thus, analytical workspace from limb 2 and limb 3 can be obtained by rotation without extra calculation from (18). The combination of the workspace boundaries of the three limbs will form the whole workspace boundaries of the 3-(rT) metamorphic parallel mechanism.

### 3.2 Singularity-free workspace with different rotation centers

Following the method in Section 4.1, some examples are given below to demonstrate the workspace boundaries with different rotation centers. The same example with that in Fig. 3 with parameters \(r_a=1, r_b=2, \alpha_a=\pi/3\) and \(\alpha_c=2\pi/9\) is used in the following. For the example in Fig. 5, the rotation center is changed from the initial configuration to \(a'_a=\pi/6\). The boundaries of limb 1 (from (20)) are shown in Fig. 5(a) where the two surfaces represent the lower and upper boundaries corresponding to the two limb input limits \(\phi_{\text{min}}=0.6, \phi_{\text{max}}=1.85\) and the space between the two blue surfaces is the workspace. By rotating the limb one boundaries by \(2\pi/3\) about the \(c_3\) axis and taking the intersection of the space between the blue surfaces, the mechanism workspace boundaries can be obtained, Fig. 5(b). By combining this workspace with its singularity loci (based on section 2.1), the singularity-free workspace is clearly seen as in Fig. 5(c). It can be seen that the mechanism workspace is separated into different parts in different orientation areas. A detailed demonstration is given in Fig. 5(c) for the workspace part in the area near the center of point \(O\) with \((c_1= c_2= c_3=0)\) in the \(O-c_1c_2c_3\) coordinate system. This is the mechanism workspace in general when the mechanism starts from the configuration of which the platform coordinate system is coincident with the base coordinate system.

When changing the rotation centers, based on the same mechanism structure parameters and limb limits, variable singularity-free workspace can be obtained. Some examples are shown in Fig. 6 in which the rotation centers are changed to (a) \(a'_a=\pi/12\), (b) \(a'_a=\pi/4\), (c) \(a'_a=\alpha'_a=0.907\) and (d)\(\alpha'_a=7\pi/18\). It can be seen that when the rotation center is under the base with \(a'_a=\pi/12\) in Fig. 6(a) the platform rotation is mainly along the \(c_3\) axis corresponding to \(z\)-axis in the mechanism coordinate. The other three cases have similar workspace shape but with different sizes in different areas.
4. CONCLUSIONS

This paper presented a unified analytical kinematics analysis of a metamorphic parallel mechanism with controllable rotation center from reconfiguration of the reconfigurable Hook joint. To cover this rotation center change, Cayley formula based geometric constraint equations were solved using Dixon’s resultant resulting in an 8th order polynomial equation which provided eight solutions for the forward kinematics analysis. Based on a new proposed coordinate system, with the three Rodrigues-Hamilton parameters in three perpendicular directions, 3D singularity loci and Limb-actuation singularity loci were analytically derived and illustrated with clear corresponding mechanism configurations. By investigating the geometric constraint equations, one of the three unknowns was expressed as a function of other two, resulting in analytic expressions for workspace boundaries. Different examples were presented to demonstrate controllable singularity-free workspace with the rotation center change.

REFERENCES