Modeling, Design & Characterization of A Novel Passive Variable Stiffness Joint (pVSJ)

Mohammad I. Awad1, IEEE Student Member, Dongming Gan1, PhD, Marco Cempini2, IEEE Member, PhD, Mario Cortese3, PhD, Nicola Vitiello34, PhD, Jorge Dias15, IEEE Senior Member, PhD, Paolo Dario3, IEEE Fellow, PhD, Lakmal Seneviratne16, IEEE Member, PhD

Abstract— In this paper we present the design and characterization of a novel Passive Variable Stiffness Joint (pVSJ). pVSJ is the proof of concept of a passive revolute joint with controllable variable stiffness. The current design is intended to be a bench-test for future development towards applications in haptic teleoperation purposed exoskeletons. The main feature of the pVSJ is its capability of varying the stiffness with infinite range based on a simple mechanical system. Moreover, the joint can rotate freely at the zero stiffness case without any limitation. The stiffness varying mechanism consists of two torsional springs, mounted with an offset from the pVSJ rotation center and coupled with the joint shaft by an idle roller. The position of the roller between the pVSJ rotation center and the spring’s center is controlled by a linear sliding actuator fitted on the chassis of the joint. The variation of the output stiffness is obtained by changing the distance from the roll-springs contact point to the joint rotation center (effective arm). If this effective arm is null, the stiffness of the joint will be zero. The stiffness increases to reach high stiffness values when the effective arm approaches its maximum value, bringing the roller close to the torsional springs’ center. The experimental results matched with the physical-based modeling of the pVSJ in terms of stiffness variation curve, stiffness dependency upon the springs’ elasticity, joint deflection and the spring’s deflection.

I. INTRODUCTION

The need of robots operating closely to humans either associating, physically interacting, or even worn by them has risen several aspects and design requirements. Although the general aspects of efficiency and robustness are taken with high importance, safety is considered one of those most important aspects. Safety should be an intrinsic feature in robots especially in the case of unexpected interactions, or sensor failures [1]. Beside the safety aspect, the interaction between the robot and the operator must show adaptability and force accuracy. This aspect has taken its importance in many applications such as rehabilitation robots, exoskeletons and haptics.

The previously mentioned aspects have motivated the development of variable impedance actuators (VIA), of which the actuator mechanical properties (inertia, damping or stiffness) affect the system equilibrium position [2]. This changes the interaction forces to adapt to different situations between the robots and the environment or users to provide safe operations and energy efficiency [2]. Based on how the impedance (stiffness and damping) is achieved, active and passive VIA concepts were proposed. In the active-by-control impedance, the behavior of a highly-reduced stiff actuator is altered via software. This concept was pioneered by DLR and adapted by KUKA [3], and it allows to adapt both stiffness and damping in a wide range and for several speeds. The disadvantages of this system lie in the high energy consumption, the need of very accurate and expensive force/torque sensors, the complexity of the control system, the incapability of storing energy and absorbing shocks [4, 31]. To overcome such drawbacks, passive compliant elements can be added to the actuator. This was early approached in the Serial Elastic Actuator (SEA) [5]. Several techniques on the SEA structures and control systems were presented in the literature [6-10]. The drawbacks of the SEA lie in the non-optimal performance and non-optimal energy efficiency. The optimal performance needs careful tuning of the joint stiffness values [11]. This motivated lots of study and new designs of variable stiffness mechanisms with passive compliance [12].

One of the easy ways to vary actuator's stiffness is by changing the spring preload. In this type, altering the perceived stiffness is achieved by changing the energy stored in the spring [31]. One evolution of this concept is the antagonistic variable stiffness actuators, where the joint stiffness is varied through the combination of two antagonistic SEAs controlled by two separate motors. Designs which fall into this category include VSA-I [6], VSA-II [13], AMASC [14], and the biological inspired joint stiffness control mechanism [15]. Other techniques were applied and several solutions were presented such as using a nonlinear connector between the output link and the spring element to adjust the preload of the linear spring shown in the MACCEPAs [16-17]. Another principle for stiffness altering is to apply a lever mechanisms between the output link and the elastic element. Varying stiffness is achieved by altering the link length between the pivot and either the elastic element or the output link. Examples of this type include the AwAS [18].
Variable stiffness actuators have been used in many applications, such as rehabilitation exoskeletons [24], surgical robotics [25], and haptics applications [26]. Applying compliant mechanisms to haptic devices has shown several advantages in reducing wear and weight [27, 28]. The disadvantages on the other hand lie in the increased control complexity [29]. To the author’s knowledge, few haptic devices which make use of compliant mechanisms have benefited from the resultant force from altering the compliant element. This force can be the key for force feedback in haptic devices [26]. Based on those, we suggest a new concept for haptic teleoperation application by using variable stiffness as a media to reflect force feedback. The mechanical joints of the master device are passive and transparent (zero stiffness) when no obstacle is touching/approaching the slave robot. On the other hand, the joints show resistive stiffness when the robot is halted or collided with an obstacle. The intermediate stiffness of the joints will be used to map the interaction forces encountered by the slave robot with the environment. By means of this approach, the force-feedback can be provided by the passive elements without direct energy consumption, thus allowing for smaller and lighter device to be developed.

In this paper, a new passive variable stiffness joint is presented as a bench-test for further development towards the above proposed haptic devices. The proof-of-concept design shows the ability to vary the stiffness from transparency to high values through changing the involvement of the elastic elements. The main feature of the presented concept is its simplicity, with the ability to achieve infinite range of stiffness. An infinite range of motion at zero stiffness is also achieved. The concept and the design are discussed in section II. Stiffness modeling is discussed in section III. While the physical implementation and experimental results are described in section IV. In section V, a discussion is conducted to compare this work with other related state-of-art designs. Finally, the conclusions are illustrated in section VI.

II. CONCEPT, MECHANICAL DESIGN

A. Concept of pVSJ

The inspiration behind the design lies in the variable lever arm mechanism but with a complete new and simpler mechanism design. In the following the concept of the pVSJ will be detailed and also compared with the other variable stiffness joints that use the lever arm mechanism.

The novelty of the pVSJ lies in the design scheme as in Fig. 1a. Two custom-made torsional springs (orange in Fig. 1a) serve the roles of both the lever and the elastic elements. The force contact point slides along the springs long arms, varying the length of the effective arm (s) of the output link where the effective arm is defined as the link between the force contact point and the joint rotation center (c). Once the output link rotates, the contact point will bend one of the two spring arms as in Fig. 1b. This will result in a contact force from the spring to the output link to balance the external torque. Change of the length of the effective arm will change the engagement of the spring with variable contact forces for the same output link rotation, indicating that the pVSJ joint can have variable stiffness. Based on this, when the force contact point coincides with the joint rotation center as in Fig. 1c the rotation of the output link will not deflect the springs since the effective arm has zero length. This results in zero joint stiffness and an unlimited rotation range. On the other hand, the highest value of stiffness is achieved by positioning the force contact point on the spring body (spring centerline n Fig. 1d). In this case, the joint can be considered as rigid with high stiffness (the structural stiffness of the spring body being the limit).

![Figure 1](image-url)
B. Mechanical Design of PVSJ

Realization of the mechanical system of the pVSJ can be seen in Fig. 2. The base (fixed) consists of a linear-actuator (a DC micromotor with planetary reducer driving a lead-screw mechanism), that drives a machined plate holding the contact-force idle roller. The contact roller slides along a diameter axis of the revolute joint, crossing the main axis of rotation. The force contact position with respect to the center of rotation (effective arm) is sensed by a linear potentiometer, which is used to close the feedback loop on the linear actuator. Two custom-made torsion springs (with mirrored design) were mounted on the shaft of the joint, with their center offset from the joint axis, and the longer arm pointing along the pivot-roller sliding direction. Only one among the two spring is enrolled when an external torque loads the system. The joint deflection is sensed by an incremental optical encoder on the top of the assembly. The force/torque sensor is mounted on the far end of the output link to measure the force/torque exerted by the user.

![Image of pVSJ principle of operation]

Figure 2: pVSJ, principle of operation: the linear actuator controls the position of the forced contact relative to the center of rotation (effective arm). When the length of effective arm=0, zero stiffness is achieved. When effective arm= max. full rigidity is achieved.

III. STIFFNESS MODELING

In order to get the model of the joint stiffness, the kinematics and the kinetics models should be presented. The kinematics is essential to estimate the deflection in the involved spring, while the kinetics model is essential to get the stiffness formula. The system works in planar coordinate (x, y) where the origin lies on the axis of rotation (z-axis) of the joint.

Deriving the kinematics (Fig. 3) of the joint starts by defining the effective arm(s), the contact roller center (C), the center of the torsional spring (Cs), the initial tangent point (B0) between the spring’s circle and its arm (at pose 0 in Fig. 3). The joint’s deflection (θ) is measured from the initial position of the spring’s center Cs0 (at pose 0 in Fig. 3) to it’s counterpart in the final position Cs (at pose 1 in Fig. 3).

![Image of Kinematics Scheme for pVSJ]

Figure 3: Kinematics Scheme for pVSJ. All points are measured from the origin (O) which is also the joint center of rotation. The initial position (pose 0) where the involved spring did not do any rotation. The final position (pose 1) is when the involved spring rotates about the origin with an angle (θ). The points are: the center of the spring at the final position (Cs), the spring’s base center the initial position (Cs0) the center of the force contact (Ci), the tangent point between the deflected spring’s base and its arm (B), and the point between the undeflected spring and its arm (B0). It is noted that the displacement between the origin and center of the force contact (s) and the joint deflection angle (θ) are the main drivers for the deflection of the involved torsional spring (β).

In general cases when s is not zero and the output link rotates an angle θ, the arm of the involved spring will form an inner tangent line to both the spring’s base circle and the force contact roller circle at points B and A respectively as in Fig. 3. In order to determine these tangential points, an algorithm to find the inner tangent of two circles (the force contact roller and the spring base) based on their center positions and radiuses was applied using the geometric relations that (CsB) ⊥ (AB) and (AB) ⊥ (ACi) (see Fig. 3). The parameters (θ and s) are involved to determine the center positions of these circles as follows:

\[
C_f = [0 \ s]^T \tag{1}
\]

\[
C_s = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} [\cos(\theta) \ -\sin(\theta)] C_{s0} \tag{2}
\]

where \(C_f\) is the center of the force contact roller, \(C_s\) is the spring’s base center at the final position (pose 1), and \(C_{s0}\) is the spring’s base center at the initial position (pose 0). The deflection angle of the involved torsion spring (β) is measured from the vector starting from the spring center C, through the initial tangent point B0(CsB0) to the vector starting from the spring center C, through the new tangent point B(CsB) through the following formula:

\[
\beta = \arccos\left(\frac{C_sB_0}{||C_sB_0||} \cdot \frac{C_sB}{||C_sB||}\right) \tag{3}
\]

From the kinematics, it can be concluded that the spring’s angular deflection (β) is a function of the effective arm length.
Figure 4: Behavior of the spring’s angular deflection ($\beta$) (color coded) with respect of the change of force contact position (horizontal-axis) and joint deflection (vertical-axis).

Exerting an external torque ($\mathbf{T}_{\text{ext}}$) on the joint will yield a deflection ($\theta$) for a given ($s$), the reaction force on the spring’s base ($\mathbf{F}_s$) has the equivalent magnitude of the force from the torsional spring’s arm on the force contact with opposite direction (see Fig. 5). Hence, it can be expressed in terms of spring’s Torque ($\mathbf{T}_s$) and the spring’s arm active length $L_s$ as follows:

$$
\|\mathbf{F}_s\| = \|\mathbf{T}_s\| = \frac{K_s \beta}{L_s}
\tag{4}
$$

where $K_s$ is the spring’s stiffness constant, at equilibrium the expression of external torque $\mathbf{T}_{\text{ext}}$ is

$$
\mathbf{T}_{\text{ext}} = \mathbf{A}_s \times \mathbf{F}_s = \mathbf{k} (\mathbf{R}_s + \mathbf{B}_s) \times \mathbf{F}_s
$$

where $\mathbf{R}_s$ is the vector with a magnitude of the maximum value of $s$. $\mathbf{B}_s$ is the vector with a magnitude of the distance between the effective arm line and the center of the involved spring, reaction force on the spring’s base, active spring arm length the vector between the origin and $\mathbf{F}_s$ and external torque respectively.

$$
K_\theta = \frac{\delta \|\mathbf{T}_{\text{ext}}\|}{\delta \theta} = K_a - K_e - K_\zeta
\tag{6}
$$

Where:

$$
K_a = K_s \frac{\beta}{L_s} \left( \frac{\|\mathbf{R}_s\| \cos(\beta)}{L_s} - \frac{\|\mathbf{B}_s\| \sin(\beta)}{L_s} \right)
\tag{7}
$$

$$
K_e = K_s \beta \left( \frac{\|\mathbf{R}_s\| \sin(\beta)}{L_s} + \frac{\|\mathbf{B}_s\| \cos(\beta)}{L_s} \right)
\tag{8}
$$

$$
K_\zeta = K_s \beta \left( \frac{\|\mathbf{B}_s\| \cos(\beta)}{L_s^2} - \frac{\|\mathbf{R}_s\| \sin(\beta)}{L_s^2} \right)
\tag{9}
$$

It can be noted from the Equations (6-9), that the joint stiffness $K_\theta$ is directly related to the stiffness $K_s$ of the springs. Fig. 7 illustrates this theoretically through showing the relation...
of the joint stiffness versus the effective arm length ($s$) for several values of available springs.

![Figure 8: Joint stiffness vs Force contact position for different values of spring’s stiffness.](image)

**IV. IMPLEMENTATION OF pVSJ**

A custom made platform that holds the force contact roller (Misumi CFFR6-16) of diameter of 16 mm is attached to the slider of a linear actuator (Misumi LX2001-B1-E2040-10). To control the effective arm length represented by the position of the contact roller a brushless DC-servomotor (Faulhaber 2232-BX4-CXD-DFF) with a 51:1 planetary gearbox (Faulhaber 22F1) is used to connect to the lead screw shaft in the linear actuator. Two torsional springs (stainless steel A316, OD: 20 mm, active coils: 4, RD: 2.5 mm, L1: 50 mm, L2= 15 mm) are fixed on the 90 mm diameter shaft of the output link. The centers of the springs have the displacements from the origin of (-18 mm, 30 mm) and (18 mm, 30 mm) respectively. This arrangement tightens the long arms of the springs on the force contact (see Fig. 6). The chassis is built from Aluminum Alloy.

Three sensors were used in the pVSJ prototype. The incremental optical encoder (Kubler 05.2420) is used to measure the joint angular deflection ($\theta$). A linear potentiometer (Curtis Wright SLS095) is used to measure the force contact’s position with respect to the center of rotation ($s$). The force/torque sensor (ATI F/T Mini-40) is used to measure the external force on the output link. National Instrument FPGA (C-RIO 9014) with National Instrument LabView® software was used for real-time control and data monitoring. Detailed design specifications can be seen in Table 1.

To validate the stiffness model, some experimental tests were conducted. By changing the values of the effective arm length ($s$), the experimental results have proven the system behaves similar to the proposed model (see Fig.8). It can be noted that the stiffness changes very slowly when the effective arm length is from (0 to 25 mm). Then the values of the stiffness boost to very high levels in the last 5 mm. This can be interpreted as the values of $L_s$ in Equations (6-9) becomes very small. On the same hand, at ($s = 30$ mm), the force contact reaches the space between the two spring bases which behave as a rigid stop.

To evaluate the ability of the pVSJ to change the stiffness, the joint deflection is measured through the optical encoder, and the external torque is measured through the Force/Torque sensor. An experiment was conducted by placing the contact roller in different positions, corresponding to different values of $s$. In each of these settings, the external torque versus the joint angular deflection for several values of $s$ was recorded and the result are illustrated in Fig. 9. It is shown that for small values of $s$, it requires low external torque for a wide span of angular deflection. As the value of $s$ increases the required external force increases for the same value of joint deflection. The span of angular deflection gets narrower by the increment of the value of $s$. This can be referred to the increment of the stiffness and the decrement of the workspace. Analyzing the experimental results for the operating range (see Fig. 10). The experimental results slightly deviates below the simulation results in the range (18 mm < $s < 20$ mm). This can be referred to the thickness of the spring arm, which was not included in the simulation model. The experimental results also deviates with maximum (0.02 rad) above the simulation results in the range (23 mm < $s < 30$ mm). This can be referred to the presence of backlash.

![Figure 6: a) pVSJ , b) disassembled pVSJ](image)

**TABLE I. pVSJ DESIGN PARAMETERS**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force contact range of motion</td>
<td>0 to 0.03</td>
<td>m</td>
</tr>
<tr>
<td>Force Contact Diameter</td>
<td>0.016</td>
<td>m</td>
</tr>
<tr>
<td>Desired range of motion for zero stiffness joint</td>
<td>$-\pi/2$ to $\pi/2$</td>
<td>rad</td>
</tr>
<tr>
<td>Desired range of motion for non-zero stiffness joint</td>
<td>$-0.27$ to $0.27$</td>
<td>rad</td>
</tr>
<tr>
<td>Spring Stiffness Constant</td>
<td>1.27</td>
<td>N.m/rad</td>
</tr>
<tr>
<td>Maximum applied torque</td>
<td>15.2</td>
<td>N.m</td>
</tr>
</tbody>
</table>
The pVSJ belongs to the category of variable-stiffness joints that alter their stiffness through changing the length of effective arm. Thanks to its design, the pVSJ can achieve zero to high stiffness (considerably rigid) with unlimited rotation in the zero stiffness case applying simple, cost-effective, easily manufactured and light-weighted components. Tuning spring effective arm was also used in the AWAS [18] but it has limited stiffness range and limited rotation range for all cases. Infinite stiffness range was achieved in AWAS-II [19] by using the concept of tuning the pivot position and a compact version, compact-VSA [20], was also developed using a rack-pinion system to control the pivot position. Based on the same model, an epicyclic gearing system was proposed in [22] to linearly position the pivot. But all those designs have limited range of rotation motion at the zero stiffness configuration due to the mechanical constraints while a large range of rotation at the zero stiffness is preferred in teleoperation as it is used when the robot does not touch the environment.

The infinite stiffness range and unlimited range of motion at zero stiffness was also achieved in the variable stiffness mechanism presented in [30]. In this mechanism, the stiffness is altered through changing the position of the supports of two leaf springs. This is realized through hypocycloid gearing mechanism which allows direct drive of the supports by the motor. However, this realization introduces back-drivability. On the other hand, the pVSJ presents a different simple implementation for altering the spring leverage, by driving the position of the force contact through a linear actuator (motor, gearbox and a leadscrew). This realization introduces velocity saturation due to the limitation in the mechanical properties of the linear actuator. This reduces the maximum rate of change in stiffness with time. However, the leadscrew is self-locking when the coefficient of friction is greater than the lead angle [32]. Moreover, using the linear torsional springs in the pVSJ eases the stiffness modeling as well as being a cost-effective option and easily manufactured. On the other hand, using the leaf springs in [30] allows the designer of obtaining a desired stiffness vs angle curve based on the design requirement. However, modeling the stiffness of the mechanism will include the non-linear Euler–Bernoulli model of the leaf spring.

VI. CONCLUSIONS

In this paper, the mechanical design, physical model and system characterization of the passive variable stiffness joint (pVSJ) were presented. The concept of varying the stiffness through altering the involvement of coil torsional springs presented here is novel. The concept is realized through a position-controlled force contact which changes the length of the effective arm representing the engagement of the springs. A physical model is presented and verified by the experimental results, which showed that the joint is capable to achieve infinite range of stiffness with infinite range of motion at the zero stiffness value.

In the future, a design underlying on the same concept will be realized, but focusing on attaining a much lower weight, a faster change for the contact roller position. These intended developments to achieve better results when applying for wearable haptic devices. The author intends to investigate the
non-linear effect of the torsional spring’s arm deflection on the stiffness performance, including more parameters in the simulation model. Finally, further investigation will be conducted to address the performance of the system in terms of safety, adaptability and energy consumption.

REFERENCES


