

Interferometric beating in digital Fourier transform holography

G.N.Oliveira^a, M.E. de Oliveira^b, J.Dias^b, and P.A.M. dos Santos*^b

^a Laboratório de Mecânica Teórica e Aplicada, Departamento de Engenharia Mecânica, Universidade Federal Fluminense, Rua Passo Pátria, 156, Niterói, R. J., Brazil, Cep.: 24.210-240; ^b Instituto de Física, Laboratório de Óptica Não-linear e Aplicada, Universidade Federal Fluminense, Av. Gal. Nilton Tavares de Souza, s/n, Gragoatá, Niterói, R. J., Brazil, Cep.: 24.210-346.

ABSTRACT

In the present work holographic interferometric beating to produce digital moiré like patterns in a well controlled digital Fourier transform holographic experiment are reported. A simple implementation of digital Fourier transform holographic setup is used to produce these patterns. This setup allows us to control some parameters like incident angle of reference beams and spatial frequency of the interferometric beating patterns. The moiré like patterns could be promising to some optical interferometric applications, including profilometry and other opto-mechanical applications.

Keywords: moiré, digital holography, Fourier, optical beating

1. INTRODUCTION

When any two regular periodic superimposed structures are illuminated, they can to produce moiré like interferometric patterns. The superposition of Ronchi rulers with slightly different superimposed pitches, for example, produces regularly spaced light fringes that have been observed and used in many optical applications [1]. Moiré patterns have been traditionally employed in optical metrology, as non-destructive testing in mechanical engineering [2,3], but optical interferometric applications based on moiré-like patterns have been extended to a many different scientific disciplines, from medical sciences[4,5] to material science[6], passing through odontology[7] and lithography process in microelectronics[8], that is, covering a lot of many different kinds of studies and applications..

In the present work we believe that, as far as we know, at first time digital Fourier transform holograms with periodic and slightly different spatial frequencies superimposed can be used to produce experimentally controlled holographic interferometric moiré like patterns

2. DIGITAL FOURIER TRANSFORM HOLOGRAPHY

Digital holography [9] in general is the method in which holograms are recorded on electronic detectors and digitally reconstructed. In the case of digital holograms the most important factors are the pixel dimensions, which allow in small areas of these devices to obtain spatial resolutions in order of at least 200 lines mm⁻¹, sufficient to make interesting holographic applications. Another significant characteristic of digital holography is the process of hologram reconstruction. In this case the use of computer specific programs, in general based on Huygens-Fresnel diffraction concepts, digitally reconstructs the object image. This process turns available more optical information about the whole system that was possible in the classical holography by making use of high-resolution photographic films. When the hologram has been formed, the complex amplitude of the object wave $H(\xi, \eta)$ can be reconstructed from the recorded plane amplitude transmission $h(x, y)$. This is made by using an expression deduced from the Fresnel-Kirchhoff integral [10], that is [11]

$$H(\xi, \eta) = \frac{i}{\lambda z} \exp\left[-\frac{i\pi}{\lambda z}(\xi^2 + \eta^2)\right] \iint_H R(x, y) h(x, y) \exp\left[-i\frac{\pi}{\lambda z}(x^2 + y^2)\right] \exp\left[i\frac{2\pi}{\lambda z}(\xi x + \eta y)\right] dx dy \quad (1)$$

*Corresponding author: pams@if.uff.br

where R is the amplitude of the reference wave, λ is the wavelength and z is the distance from the object to the recording plane. The intensity $I(\xi, \eta)$ and the phase $\varphi(\xi, \eta)$ of the scene is available through the complex amplitude of the object wave $H(\xi, \eta)$ and can be obtained by

$$I(\xi, \eta) = |H(\xi, \eta)|^2, \quad \varphi(\xi, \eta) = \arctan \frac{\text{Im}[H(\xi, \eta)]}{\text{Re}[H(\xi, \eta)]} \quad (2)$$

The digital hologram of a Fourier transform of light beam diffracted from the object is obtained when the distance from the plane containing both the object and the reference light source is far from the hologram plane. In the recording configuration of lensless Fourier holography, the spherical phase factor associated with the Fresnel diffraction pattern of the object is eliminated by use of a spherical reference wave [11], with the same average curvature of the recorded plane, then Eq.(1) is modified to [12]

$$H(\xi, \eta) = -\frac{i|R|}{\lambda z} \exp\left[-\frac{i\pi}{\lambda z}(\xi^2 + \eta^2)\right] \iint_H h(x, y) \exp\left[i\frac{2\pi}{\lambda z}(\xi x + \eta y)\right] dx dy \quad (3)$$

Therefore, the calculation is effectively done using the fast Fourier transform (FFT) algorithm. The factor in front of the sum is affecting the phase and can be neglected for some applications [6]. Therefore, the propagation of the beams falls in the Fraunhofer diffraction limit. This is the basis of Digital Fourier Transform Holography (DFTH) method. In this case, the amplitude reconstruction in Eq. (2) could be described by a pure Fourier transform equation, adapted to a CCD as recording sensor of N^2 pixels of $\Delta x \Delta y$ area. The recorded intensity pattern is discretized in intervals of $N\Delta x$ and $N\Delta y$ [6], where (m, n) means the index that corresponds to each pixel of the discretized image. Then, the image reconstruction equation is expressed by [12]

$$H(m, n) = i \frac{|R|}{\lambda z} \left[\sum_{k=0}^{N-1} \sum_{l=0}^{N-1} h(k, l) \exp\left[-i2\pi\left(\frac{km}{N} + \frac{ln}{N}\right)\right] \right] \quad (4)$$

where the phase factor is absent.

3. EXPERIMENTAL SETUP

In the present work moiré patterns have been produced by DFTH method. Moiré patterns can be obtained by the superposition of two or more sinusoidal gratings with slightly different spatial frequencies. In order to produce the digital moiré patterns, a simple implementation of digital Fourier transform holographic setup is proposed. Usually in the holographic setup for DFTH both the reference light source and the object are in the same plane at distance z to find the physical conditions imposed into Eq. (2). However, in the alternative holographic configuration, illustrated in Fig. 1, the necessity of a plane containing both the object and reference light source at distance z is replaced by a virtual reference light source from the expander E. In this way, the proposed configuration reproduces the same optical propagation conditions to obtain the DFTH hologram.

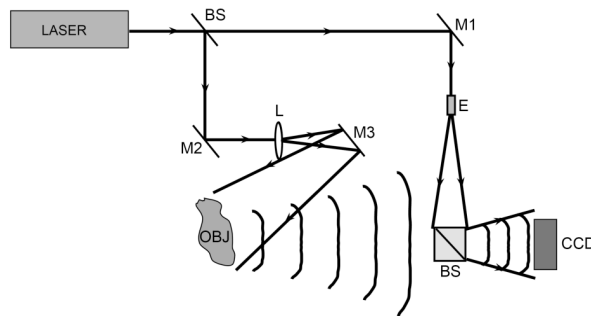


Fig. 1. The sketch of proposed holographic setup for shear testing using DFTH.

In Fig. 1, M1 and M2 are mirrors, M3 is a mirror controlled by computer, BS are beam splitters, E is an expander and L a lens. The CCD camera (Sony model XCD-SX910) used to record the images of the object has a resolution of 1376 x 1024 pixels and the pixel of the CCD camera has the area of approximated $4.65 \times 4.65 \mu\text{m}^2$. In this configuration, the Fig. 2 illustrates the image reconstruction of the DFTH hologram of a dice, as a simple example. This image was obtained using the proposed holographic setup in Fig. 1. The dice appears in two reconstructed images, one inverted, in both sides of zero order. This is an expected image reconstruction of a Fourier transform hologram.

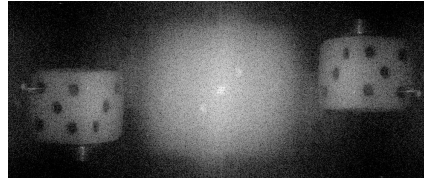


Fig. 2. Image reconstruction of the Fourier transform hologram of a simple object made with the configuration shown in Fig. 1

This configuration has two great advantages. The first one is the control of the incidence angle between object beam and reference beam on the surface of the CCD. And the second one is the control of the object illumination. Fig. 3 shows three images up to down, where the reconstructed images of the object, a little aluminium plate, in both sides of zero order, appear with different distances made with the angles $\theta_1 = 0.25$, $\theta_2 = 0.50$ and $\theta_3 = 0.75$ degrees. Then, the incident angle of beams allows the controlling of this distance. By rotating and shifting the BS (see Fig. 1) the incidence angle increases or decreases, thus enabling to control this experimental parameter.

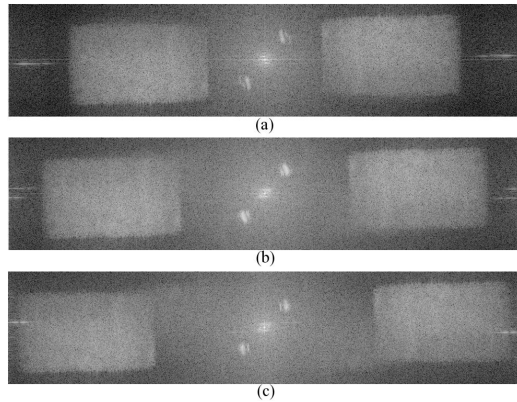


Fig. 3. Three images reconstructed from the Fourier transform holograms with three different incidence angles

The second one is to have a possibility to rotate the mirror M3, that allows to change the object illumination. This angle rotation is made around the orthogonal direction to the main axis one, that contains the object, the BS and the CCD camera, that is, in the incidence plane.

4. RESULTS AND DISCUSSION

In order to produce the digital holographic moiré patterns, the M3 mirror rotation is carefully controlled by a step motor connected to a computer. Firstly, the first hologram is recorded. After that, the mirror M3 is rotated by a little angle and then the second hologram is recorded. These two holograms, with slightly different phase, resulting of different illuminations, are produced. So, the two holograms recorded this way and represented by

$$\begin{aligned} h_1(x,y) &= h_0(x,y) \cdot \text{sen}(k_1x) \\ h_2(x,y) &= h_0(x,y) \cdot \text{sen}(k_2x) \end{aligned} \tag{5}$$

where k_1 and k_2 are the spatial frequencies, are added, and after a simple algebraic manipulation give as result

$$h(x,y) = 2h_0(x,y) \cos(k^m x) \text{sen}(\bar{k} x) \quad (6)$$

that is the resulting hologram. In this result $\bar{k} = (k_1 + k_2)/2$ is the average spacial frequency and $k^m = (k_1 - k_2)/2$ is the modulation spatial frequency.

The image reconstruction now is made using Eq. (4). So, the resulting hologram from

$$H(m,n) = i \frac{|R|}{\lambda Z} \left[\sum_{k=0}^{N-1} \sum_{l=0}^{N-1} [h_1(k,l) + h_2(k,l)] \exp \left[-i2\pi \left(\frac{km}{N} + \frac{ln}{N} \right) \right] \right] \quad (7)$$

using the result of Eq. (6), is calculated. The intensity of the reconstructed images in Fig. 4 shows digital moiré like patterns obtained like an optical *beat* phenomenon, onto the object images for three different modulation frequencies k^m . The control of this spacing is made increasing or decreasing the angle of mirror M3. In Fig. 4, up to down, these angles are $\phi_1 = 0.19$, $\phi_2 = 0.42$ and $\phi_3 = 0.52$ degrees.

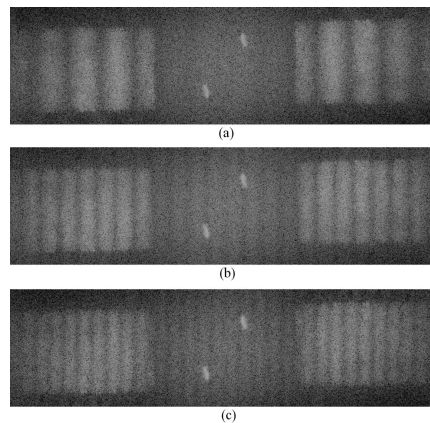


Fig. 4. Intensity of reconstrated image and digital moiré like patterns produced for three different modulation frequencies.

Moreover, the phase $\varphi(\xi, \eta)$ associated to the reconstructed images shows the aspect of the predicted beat phenomenon. Using the phase definition in Eq. (2), after an operation of unwrapping, the phase, corresponding to the reconstructed intensities shown in Fig.4 up to down, was obtained, and is shown in Fig. 5, also up to down, respectively

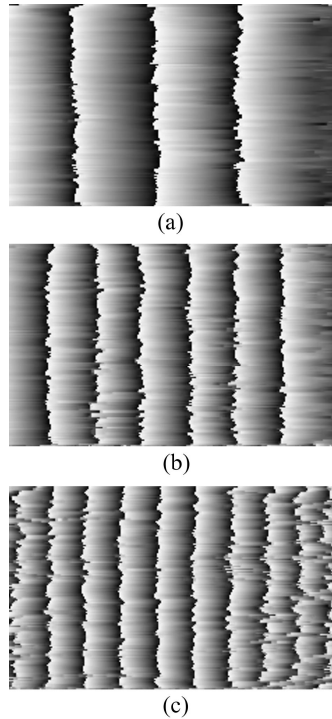


Fig. 5. Corresponding phase maps for the reconstructed image of Fig.4, respectively, for three different modulation rates.

5. CONCLUSION

In the present paper we have demonstrated, as far as we know, at first time that digital Fourier transform holograms, with approximately periodic and slightly different spatial frequencies, produce holographic interferometric moiré like patterns. In our experimental holographic setup, not only the incident angle is controlled, but also the direction of the object illumination beam to effectively produce the moiré like patterns by a beat phenomenon, in both intensity and phase, by digital Fourier transform holography. We believe that due to the great advance experienced by digital holography in terms of scientific and industrial applications our proposal could be interesting mainly in mechanical engineering, i.e., fracture detection in materials surface, quality inspection in metallic structures, profilometry and interferometric non destructive testing for general purposes.

Acknowledgements. We would like to thank to Dr. I. Costa for the great help in text spelling and grammar corrections, and to the Brazilian financial support agencies CNPq (Conselho Nacional de Pesquisa), CAPES (Cordenadoria de Aperfeiçoamento de Pessoal de Nível Superior) and FAPERJ (Fundação Carlos Chagas Filho de Apoio a Pesquisa do Estado do Rio de Janeiro).

REFERENCES

- [1] Indebetouw, G.J. and Czarnek, R., "Selected Papers on Optical Moiré and Applications," SPIE Ed. Books (1992)
- [2] Dally, J.W. and Riley, W.F., "Experimental Stress Analysis," McGraw Hill Kogakusha, (1978).
- [3] Post, D., Han, B. and Ifiju, P., "High Sensitivity Moire Experimental Analysis for Mechanics," Springer-Verlag, (1994).

- [4] Kim, H.S., Ishikawa, S., Ohtsuka, Y., Shimizu, H., Shinomiya, T. and Viergever, M.A., "Automatic scoliosis detection based on local centroids evaluation on Moiré topographic images of human backs," *IEEE Trans. Med. Imaging* 20, 1314 (2001).
- [5] Munoz, P. and Alda, J., "Visual perception of the Moiré effect," *OphthalmicPhysiol.* 19, 427 (1999).
- [6] Fujiwara, E., Irie, S., Nemoto, T., Isoda, S. and Kobayashi, T., "A scanning tunneling microscopy study on the monolayer epitaxy of [cyano(ethoxycarbonyl) methylene]-2-ylidene-4,5-dimethyl-1,3-dithiole on graphite and its dynamic feature," *Surface Sci.* 459, 390 (2000).
- [7] Kishen, A. and Asundi, A., "Investigations of thermal property gradients in the human dentine," *J. Biomed. Mater. Res.* 55, 121 (2001).
- [8] Semaltianos, N.G., Scott, K. and Wilson, E.G., "Electron beam lithography of Moiré patterns," *Microelect. Eng.* 56, 233 (2001).
- [9] Schnars, U. and Juptner, W., "Direct recording of holograms by a CCD target and numerical Reconstruction," *Appl. Opt.* 33, 179 (1994).
- [10] Collier, R.J., Burckhardt, C.B. and Lin, L.H., "Optical Holography," Academic Press, (1971).
- [11] Schnars, U. and Juptner, W., "Digital recording and numerical reconstruction of holograms," *Meas. Sci. Technol.* 13, 85 (2002).
- [12] Dirksen, D., Drost, H., Kemper, B., Delere, H., Deiwick, M., Sheld, H. and von Bally, G., "Lensless Fourier holography for digital holographic interferometry on biological samples," *Opt. Lasers Eng.* 36, 241 (2001).