International Journal of Control, Automation and Systems 14(5) (2016) 1-11
http://dx.doi.org/10.1007/s12555-015-0171-3
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Adaptive Control for Robot Manipulators using Multiple Parameter Models

Shafiqul Islam*, Peter X. Liu, Jorge Dias, and Lakmal D. Seneviratne

Abstract: In this paper, we propose multiple parameter models based adaptive switching control system for robot manipulators. We first uniformly distribute the parameter set into a finite number of smaller compact subsets. Then, distributed candidate controllers are designed for each of these smaller compact subsets. Using Lyapunov inequality, a candidate controller is identified from the finite set of distributed candidate controllers that best estimates the plant at each instant of time. The design reduced the observer-controller gains by reducing modeling errors and uncertainties via identifying an appropriate control/model via choosing largest guaranteed decrease in the value of the Lyapunov function energy function. Compared with CE based CAC design, the proposed design requires smaller observer-controller gains to ensure stability and tracking performance in the presence of large-scale modeling errors and disturbance uncertainties. In contrast with existing adaptive method, multiple model based distributed hybrid design can be used to reduce the energy consumption of the industrial robotic manipulator for large scale industrial automation by reducing actuator input energy. Finally, the proposed hybrid adaptive control design is experimentally tested on a 3-DOF Phantom$^{\textbf{TM}}$ robot manipulator to demonstrate the theoretical development for real-time applications.

Keywords: Adaptive switching control, Lyapunov method, multiple models, observer.

1. INTRODUCTION

Last two decades, *certainty equivalent (CE)* principle based single model adaptive control technique for nonlinear systems has been studied by many researchers [1–5]. It has been proven through Lyapunov analysis that the single model based adaptive approach can achieve asymptotic tracking under certain parameter variations by assuming that the transient tracking errors are chosen according to designer desired specification. However, the single model based classical adaptive control paradigm can only ensure asymptotic convergence as the transient tracking property does not consider in the stability analysis. On the other hand, when the parameters and initial conditions changes in large magnitude, classical adaptive design either for the state or output feedback may exhibit large transient tracking errors due to the presence of the large modeling errors uncertainty. This may be because the classical design is based on the assumption that uncertain parameters are appeared linearly with respect to unknown nonlinear system dynamics. For example, the asymptotic tracking performance of classical adaptive control system design for robot manipulators relies on the strict linear dynamical properties of the system, e.g., [1, 3, 5–18], and many others) where uncertain parameters of the system model and operating payload parameters are assumed to be appeared linearly with respect to unknown nonlinear system dynamics. This means that the system requires to satisfy linear dynamical properties, such as, inertial matrix is symmetric, bounded, and positive definite inequalities, the norm of the gravity and centripetal-coriolis forces are upper bounded. To improve the tracking performance against large modeling error uncertainty, one can use high controller-observer gains to increase the convergence rate of the state and parameter estimates. However, the demand of high observer-controller gains makes the CE-based adaptive approach very difficult to realize in practical applications as the control efforts/control gains in most nonlinear control systems are limited. To improve the tracking performance and reduce the observer-controller gains from CE-based classical adaptive control (CAC) paradigm, we propose to employ multiple parameter models based adaptive switching control method for robot manipulators. Such control method can be viewed as a special kind of feedback control technique that supervises the plant to meet the desired objective by switching

Manuscript received April 22, 2015; revised August 10, 2015; accepted October 6, 2015. Recommended by Associate Editor Changchun Hua under the direction of Editor Myotaeg Lim.

Shafiqul Islam is with University of Ottawa, K1N 6N5 Canada, and KUSTAR, Abu Dhabi, UAE (e-mail: shafiqul11975@gmail.com). Peter X. Liu is with Carleton University, K1S 5B6 Canada (e-mail: xpliu@sce.carleton.ca). Jorge Dias is with University of Coimbra, Portugal, and KUSTAR, Abu Dhabi, UAE (e-mail: jorge@deec.uc.pt). Lakmal D. Seneviratne is with Kings College London, UK, and KUSTAR, Abu Dhabi, UAE (e-mail: lakmal.Seneviratne@kcl.ac.uk).
* Corresponding author.

an appropriate model from multiple plant parameter models. The idea of switching between multiple parameter models was used in the past in [19] for a class of linear system to deal with stabilization problem of the estimated model in indirect control framework. This method was then applied in [20–24] in model reference and certainty equivalent principle based classical adaptive control framework provided that the relative degree of the plant was unknown. Authors in [25–27] also presented switching supervisory control in adaptive framework extending classical gain scheduling method without *a priori* information about the plant dynamics. Earlier contribution on switching control method can be found in [28–31]. Interested readers are referred in [22] to get overview of adaptive switching control designs. The one-DOF and two-DOF switching control design in continuous time domain was extensively studied in [24] and [32]. To show that the switching among multiple models can improve the performance in adaptive system, authors in [33–35] developed adaptive control using multiple models for a class of linear system. The switching was based on using a performance index designed by output prediction errors with the help of quadratic optimal control to select the candidate controller from finite set of candidates at every instant. Interesting unfalsified control framework was extensively studied by Safonov and his group in [36–39] as a switching criterion to select an appropriate candidate controller from a family of controllers. These model-free adaptive switching designs can guarantee the stability of the time-invariant system in the presence of unmodeled dynamics, disturbances and large parametric uncertainties. Recently, authors in [40–42] have introduced new types of multi-model unfalsified adaptive switching technique by combining multi-model and unfalsified adaptive switching control mechanism. To deal with persistent time variation in plant dynamics, authors in [40, 43–47] extended multimodel unfalsified adaptive switching method. They have showed that the existing multi-model unfalsified adaptive switching can be used to deal with the persistent plant variations. In view of the control structure, we can notice that many of these above mentioned switching schemes are designed by assuming that all the states are available for measurement. In this work, we introduce multiple models based adaptive switching control algorithm for multi-input multi-output robotic system. The main purpose of using adaptive switching scheme is to improve the transient tracking performance and to reduce the control efforts by using smaller values of learning and linear control gains. We introduce two types of switching schemes as pre-routed and instantaneous switching logic for supervisor to identify an appropriate controller from a finite set of candidate controller at each instant of time. For pre-routed control, first local candidate controllers are developed and ordered them arbitrarily in series. Then, during pre-fixed period of time, the pre-defined resetting condition designed by the time derivative of candidate Lyapunov function is employed to select a controller from the controller sets. However, the pre-routed switching method may generate large transient tracking performance in the presence of large number of candidate controllers resulting large transient performance and control efforts. To deal with this problem, we then introduce Instantaneous switching logic with the largest decrease in the value of candidate Lyapunov functions as resetting condition at each instant of time. We first distribute uncertain parameter sets into a finite number of smaller subsets and then design candidate controller corresponding to each of these distributed candidate sets. The candidate controllers are placed in parallel and switch instantaneously with specified dwell-time constant based on using largest guaranteed decrease in the value of the Lyapunov function. Compared with reported results, the proposed design can be applied for both full state and partial state measurement for multi-input multi-output system. The design can be used to improve the transient tracking performance and reduces the observer and controller gains/control efforts with smaller values of observer-controller gains from CAC design by allowing its parameter estimate to be reset into a finite set of distributed candidate models. Experimental results are presented to illustrate the theoretical development of the proposed method for practical applications. The rest of the paper is organized as follows: Section 2 introduces distributed adaptive switching control algorithms. Section 3 presents experimental results on real robot manipulators to demonstrate the effectiveness of the proposed design. Finally, Section 4 concludes the paper.

2. ADAPTIVE SWITCHING CONTROL DESIGN

In this section, we first analyze the difficulty associated with the CE principle based single model adaptive control paradigm for nonlinear mechanical systems. To do that, let us design CE based adaptive control algorithm for robot manipulator whose motion dynamics can be written as follows [1, 5, 17]:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau, \tag{1}$$

where $q\in\Re^n$ is the position vector, $\dot{q}\in\Re^n$ is the velocity vector, $\ddot{q}\in\Re^n$ is the acceleration vector, $\tau\in\Re^n$ is the input torque, $M(q)\in\Re^{n\times n}$ is the symmetric and uniformly positive definite inertia matrix, $C(q,\dot{q})\dot{q}\in\Re^n$ is the Coriolis and centrifugal loading vector, and $G(q)\in\Re^n$ is the gravitational loading vector. Then, we can represent the system (1) in the error state space form as follows

$$\dot{e}_1=e_2, \dot{e}_2=\phi_1(e)+\phi_2(e_1)\tau-\ddot{q}_d, \tag{2}$$

where $e_1=(q-q_d)$, $e_2=(\dot{q}-\dot{q}_d)$, $\phi_1(e)=-M^{-1}(e_1+q_d)[C(e_1+q_d,e_2+\dot{q}_d)(e_2+\dot{q}_d)+G(e_1+q_d)]$ and $\phi_2(e_1)=M^{-1}(e_1+q_d)$. We now analyze the tracking property of CAC on the given system model (2). The stability property of CAC approach will be based on the well-known dynamical properties of the system (1) [48]. The control objective is that the position $q(t)$ asymptotically tracks the desired position $q_d(t)$. We assume that $q_d(t)$, its first and second derivatives are bounded such that $Q_d = [q_d \ \ \dot{q}_d \ \ \ddot{q}_d]^T$ with Q_d be given any compact set $Q_d \in \Omega_d \subset \Re^{3n}$. To meet this control objective, many single models CE principle based classical adaptive control laws have been reported in the literature. We consider the following CE principle based classical adaptive control law [5] as $\tau(e, Q_d, \hat{\theta}) = Y(e, \dot{q}_d, \ddot{q}_d)\hat{\theta} - K_P e_1 - K_D e_2$ with $\dot{\hat{\theta}} = -\Gamma Y^T(e, \dot{q}_d, \ddot{q}_d)S$ where $Y(e, \dot{q}_d, \ddot{q}_d)\hat{\theta} = \hat{M}(q)\ddot{q}_d + \hat{C}(q, \dot{q}_r)\dot{q}_d + \hat{G}(q)$, $K_P \in \Re^{n\times n}$, $K_D \in \Re^{n\times n}$, $S = (e_2 + \lambda e_1)$, $\dot{q}_r = (\dot{q}_2 - \lambda e_1)$ with $q_2 = \dot{q}$, $\lambda = \frac{\lambda_0}{1+\|e_1\|}$, $\lambda_0 > 0$, $\Gamma \in \Re^{m\times m}$ positive definite constant diagonal matrices and $\hat{M}$, $\hat{C}(.)$ and $\hat{G}(.)$ define the estimates of the $M(.)$, $C(.)$ and $G(.)$, respectively. The adaptation mechanism is used to cope with parametric uncertainty. Notice from the adaptive design that uncertain model parameters $\theta \in \Re^{m\times 1}$ assumed to be appeared linearly with respect to nonlinear functions $Y(e, \dot{q}_d, \ddot{q}_d)$. The learning estimates may exhibit discontinuous property even after the parameter estimate converges to actual one. To avoid such a parameter drift, the estimates $\hat{\theta}$ can be adjusted with the strict parameter projection condition [49–51]. This adaptive controller design is designed by using the Lyapunov function as [5] $V(e, \tilde{\theta}) = \frac{1}{2}S^T MS + \frac{1}{2}e_1^T K_P e_1 + \frac{1}{2}\tilde{\theta}\Gamma^{-1}\tilde{\theta}$ with $\tilde{\theta} = (\hat{\theta} - \theta)$. The time derivative $\dot{V}(e, \tilde{\theta})$ along with the closed loop trajectories can be simplified to obtain asymptotic stability condition as $\dot{V}(e, \tilde{\theta}) \leq -\lambda_{min}(\Pi)\|e\|^2$ $\forall \hat{\theta}(0) \in \Omega$, $\forall \theta(0) \in \Omega$, $\forall \hat{\theta} \in \Omega_\delta$, $\forall e(0) \in \Omega_{co}$, $\forall e \in \Omega_c$ with $\Omega_c = \{e \mid e^T Q_{sm} e \leq c\}$, $c > 0$ and $Q_{sm} = \begin{bmatrix} 0.5M & 0.5M\lambda \\ 0.5M\lambda & 0.5\left(\lambda^2 + K_P\right) \end{bmatrix}$, $\Pi = \Theta^T \Delta \Theta$ with Δ and Θ defined as $\Delta = \begin{bmatrix} K_1 I & 0 \\ 0 & K_2 I \end{bmatrix}$ and $\Theta = \begin{bmatrix} \frac{\lambda I}{2} & I \\ \frac{\lambda I}{2} & 0 \end{bmatrix}$ where $K_1 = [\lambda_{min.}(K_D) - 3\lambda_0 M_M - 2\lambda_0 C_M]$ and $K_2 = \left[\frac{4\lambda_{min.}K_P}{\lambda_0} - \lambda_{max.}(K_D) - 2\lambda_0 M_M - 2\lambda_0 C_M\right]$. To implement the state feedback based controller, it is assumed that the system output and its derivatives are available for feedback. We now consider the practical situation where velocity sensors are not available. This means that the state vector e in the above control law is not available for measurement and required to be estimated by a suitable estimator $\hat{e}$. If e is replaced by an estimate $\hat{e}$, then one obtains single model adaptive output feedback control law $\tau(\hat{e}, Q_d, \hat{\theta})$ as $\tau(\hat{e}, Q_d, \hat{\theta}) = Y(\hat{e}, \dot{q}_d, \ddot{q}_d)\hat{\theta} - K_P\hat{e}_1 - K_D\hat{e}_2$ where $\hat{\dot{q}}_r = (\hat{e}_2 + \dot{q}_d - \lambda\hat{e}_1)$ and $\hat{\theta}$ is replaced by the following linear estimator [49], $\hat{e}$, as

$$\dot{\hat{e}}_1 = \hat{e}_2 + \frac{H_1}{\varepsilon}\tilde{e}_1, \dot{\hat{e}}_2 = \frac{H_2}{\varepsilon^2}\tilde{e}_1, \tag{3}$$

where $\tilde{e}_1 = (e_1 - \hat{e}_1)$, ε is a small constant design parameters needs to be specified, H_1 and H_2 are chosen such that the matrix $\begin{bmatrix} -H_1 & I \\ -H_2 & 0_{n\times n} \end{bmatrix}$ is Hurwitz. The idea in output feedback convergence analysis is to show that the performance achieved under adaptive state feedback $\tau(e, Q_d, \hat{\theta})$ can be recovered by using output feedback design $\tau((e - \zeta(\varepsilon)\eta), Q_d, \hat{\theta})$ where $\eta = \left[\eta_1^T, \eta_2^T\right]^T$, $\eta_1 = \frac{\tilde{e}_1}{\varepsilon}$, $\eta_2 = e_2 - \hat{e}_2 = \tilde{e}_2$, $\varepsilon\dot{\eta}_1 = \eta_2 - H_1\eta_1$, $\varepsilon\dot{\eta}_2 = \varepsilon[-\ddot{q}_d + \phi_1(e) + \phi_2(e_1)\tau(\hat{e}, Q_d, \hat{\theta})] - H_2\eta_1$, $\zeta(\varepsilon) = \begin{bmatrix} \varepsilon I_{n\times n} & 0_{n\times n} \\ 0 & I_{n\times n} \end{bmatrix}$, $\hat{e} = e - \zeta(\varepsilon)\eta$. This means that, for any given $\forall e(0) \in \Omega_{co} \subseteq \Omega_c$, $\forall \hat{e}(0) \in \Omega_{co}$, $\forall \hat{\theta}(0) \in \Omega$ and $\forall \theta(0) \in \Omega$, there exists a small observer design constant ε_1^* such that for all $0 < \varepsilon < \varepsilon_1^*$, all the state variables of the closed loop system are bounded and their bounds can be made closed to the bound achieved under state feedback design by using small value of ε as $\dot{V} \leq -\lambda_{min.}(\Pi)\|e\|^2 + \chi\varepsilon$ with $\chi > 0$. The stability property for adaptive state and output feedback system is achieved by assuming that there exists very high observer-controller gains provided that parameter θ remains bounded by strict projection mechanism. The problem with CAC design is that the design exhibits poor transient tracking response in the presence of large parametric errors uncertainty. One may employ high values of learning and observer gains in order to achieve desired transient tracking performance. The main practical problem, however, is that the observer-controller gains requires to increase with the increase of the parametric uncertainty resulting very large control efforts. To deal with the problem associated with high observer-controller gains, we introduce multiple parameter models based adaptive control approach that can be used to reduce the value of observer-controller gains. First, we consider that the plant parameters θ are unknown but belongs to a known compact set Ω. We then equally distribute the parameter set Ω into a finite number of smaller candidate subsets such that $\theta_i \in \Omega_i$ with $\Omega = \bigcup_{i=1}^N \Omega_i$ and $\theta \in \Omega_i$. Then, for a given compact set of the initial conditions of interest $e(0) \in \Omega_{co}$, we design candidate controller correspond to each of these smaller subsets as

$$\tau^i(e, Q_d, \theta_i) = Y(e, \dot{q}_d, \ddot{q}_d)\theta_i - K_P e_1 - K_D e_2 \tag{4}$$

with $(\theta, \theta_i) \in \Omega_i$, such that for every $\theta \in \Omega_i$ all the signals in the closed loop system formulated by (2) and (4) started inside the initial set Ω_{co} are bounded. We now assume that the system does not have velocity sensors in order to reduce the weight and cost of the system. To reproduce unknown velocity signals, we replace e by the output of the linear estimator (3). Then, the velocity independent adaptive controllers can be designed as

$$\tau^i(\hat{e}, Q_d, \theta_i) = Y(\hat{e}, \dot{q}_d, \ddot{q}_d)\theta_i - K_P\hat{e}_1 - K_D\hat{e}_2. \tag{5}$$

The control gains K_P and K_D are common to all the candidate controllers N. To identify an appropriate controller from candidates $\tau^i(e, Q_d, \theta_i)$, we use pre-ordered control-switching logic where the inequality for the time derivative of the Lyapunov candidate as $\dot{V}(e, \tilde{\theta}) \leq -\lambda_{min}(\Pi)\|e\|^2$ is used as a control switching condition. Now, for the identification of the best candidate controller from (5) with observer (3), our first task is to ensure the robust reconstruction of unknown velocity state vectors. Notice from the Lyapunov inequality $\dot{V} \leq -\lambda_{min.}(\Pi)\|e\|^2 + \chi\varepsilon$ that one cannot make state estimation errors to zero as $\varepsilon \neq 0$. For a given observer design constant ε, there exists a short transient period such that the state estimates $\hat{e}$ decay exponential fast to a small compact set Ω_ε. The short transient peaking time $T_1(\varepsilon)$ can be determined as $T_1(\varepsilon) = \frac{\varepsilon}{\gamma} \ln\left(\frac{k_o}{\beta\varepsilon^4}\right)$ where ε is a small constant, $k_o = k^2\lambda_{max.}(P) = \frac{k^2}{2\gamma}$, $\gamma = \frac{1}{2\lambda_{max.}(P)}$, $e(0) - \hat{e}(0) \leq k$ with $k \geq 0$, $\|[-\ddot{q}_d + \phi_1(e) + \phi_2(e_1)\tau^s((e - \zeta(\varepsilon)\eta), Q_d, \hat{\theta})]\| \leq k_1$ for $k_1 > 0$, $\beta = 16\|P\|^3 k_1^2$, $\|P\| = \lambda_{max.}(P)$, P is the solution of the Lyapunov-equation as $PA_o + A_o^T P = -I$ with $A_0 = [-H_1\ I; -H_2\ 0_{n\times n}]$ for all $t \in [T_1(\varepsilon), T_3]$. After this transient peaking time, the estimation error converge to a small value, namely $O(\varepsilon)$, that closed to zero. To ensure that, the short time t_d requires to select such that $T_1(\varepsilon) < t_d$. Then, we use $\dot{V} + \lambda_{min}(\Pi)\|e\|^2 \leq k_f$ with $k_f = \chi\varepsilon$ as a control switching inequality for the output feedback based adaptive control (5). Based on our above analysis, we can state the following pre-routed control-logic [40–42, 52, 53].

Algorithm 1: Suppose that the controller index $i \in \mathcal{M}$ is acting in the loop at time t. Then, we follow the following control-logic to identify a controller that satisfies the pre-specified Lyapunov inequality $\dot{V} \leq -\lambda_{min.}(\Pi)\|e\|^2 + k_f$.

A. We consider the small time $t_o = 0$, controller index $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$ and a dwell time $t_d > T_1(\varepsilon)$.

B. Then, we put the algorithm (5), $\tau(\hat{e}, Q_d, \hat{\theta})$ with $\hat{\theta}$ is provided by standard adaptation law (the output of $\dot{\hat{\theta}} = -\Gamma Y^T(\hat{e}, \dot{q}_d, \ddot{q}_d)\hat{S}$) with observer (3), in the loop and dwell it for a short period of time $t \in [t_o, t_o + t_d]$.

C. For $t \geq t_o + t_d$, we check the pre-specified inequality using with the derivative of the Lyapunov-function candidate $\dot{V}(t) \leq k_f$. If the inequality satisfies, then we stay with the classical control in the loop. If not, then we put the first candidate controller, $\tau^i(\hat{e}, Q_d, \theta_i)$, with $i = 1$.

D. We again dwell this controller for small time $t_o + t_d$ with $t_o = t_i$ and monitor the inequality for the derivative of the Lyapunov function to see whether or not the function is decreasing in order to switch to the next candidate controller. If the controller does not satisfy the inequality, then we switch again to the next candidate, $\tau^i(\hat{e}, Q_d, \theta_i)$, with $i = 2$.

E. We continue to repeat the search for the candidates until we find a controller that satisfies the derivative of the Lyapunov inequality. Using Algorithm 1, we now state the main results in the following Theorem 1.

Theorem 1: Consider the closed loop control systems designed by using (2), (3) and (5) under the control switching logic defined in Algorithm 1 with the resetting inequality for the derivative of the Lyapunov-function $\dot{V}(t) \leq k_f$. Then, for any given $(e(0), \hat{e}(0)) \in \Omega_{co}$, $\theta \in \Omega_i$ and $\theta_i \in \Omega_i$ with $i \in \mathcal{M}$, there exists $\varepsilon > 0$ and $t_d > T_1(\varepsilon)$ such that the candidate controller, corresponding to an appropriate distributed model, according to the Algorithm 1, is tuned to the plant which ensures that all the state variables of the closed loop system are bounded.

Proof: The proof of Theorem 1 can be shown along the logic defined in Algorithm 1. For the sake of simplicity, we remove the details proof which can be illustrated along the line of the method introduced in [52, 53]. The switching Algorithm 1 may produce unacceptable transient tracking performance [52, 53] in the face of large number of candidate controllers results in long period of the switching search. This may be because of the presence of large number of candidate controllers which may require the logic to travel through all candidates before converging to the one that satisfies the switching inequality. On the other hand, if the parameter changes after switching events, then the logic stated in Algorithm 1 will be insensitive to the parameter change which may cause large control efforts and transient tracking performance. To avoid transient tracking from Algorithm 1, let us allow the parameter estimates to be reset instantaneously using with the following switching Algorithm 2. We first consider that, for a given compact set of initial conditions of interest $e(0) \in \Omega_{co}$ and for every $\theta \in \Omega_i$, with $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$, we design a set of candidate controllers $\tau^i(\hat{e}, Q_d, \theta_i)$ such that, for every $\theta \in \Omega_i$, all the signals in the closed loop system started inside Ω_{co} are bounded such that the error trajectories are asymptotically stable. We also consider that a set of Lyapunov-function candidates corresponding to a set of candidate controllers such that $\alpha_2^i\|e\|^2 \leq V_i(e, \tilde{\theta}_i) \leq \alpha_3^i\|e\|^2\ \forall e \in \Omega_c^i = \{(e, \tilde{\theta}_i) \mid V_i(e, \tilde{\theta}_i) \leq c\}$ and $\forall(\theta, \theta_i) \in \Omega_i$ where $c > 0$, $\tilde{\theta}_i = (\theta_i - \theta)$ and α_2^i and α_3^i are bounded positive constants. We then follow the following logic to identify a controller corresponding to a model which closely approximates the plant at each instant of time such that the error trajectories are bounded and asymptotically stable.

Algorithm 2: Suppose that the candidate controllers $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$ and the candidate Lyapunov-functions, $V_i(e, \tilde{\theta}_i)$, are available at any time t. Then, we apply the following switching-logic to identify a controller which closely estimates the plant.

A. Let us define the initial time $t_o = 0$, the switching index $i \in \mathcal{M} = \{1, 2, 3, ..., N\}$ and a small positive time $t_d > 0$.

B. We then put the controller $\tau(e, Q_d, \hat{\theta})$ (4) with standard adaptation mechanism for a short period of time $t \in [t_o, t_o + t_d]$.

C. For $t \geq t_o + t_d$, we monitor the inequality for the Lyapunov-function candidates to see which candidate provides largest decrease in the value of $\triangle W_i(t) = V_i(t_s) - V_0(t) \leq 0$ where $t_s \geq t_o + t_d$ is the switching time and $V_0(t) = \frac{1}{2}e^T(t)Q_{sm}e(t) + \frac{1}{2}\tilde{\theta}^T(t)\Gamma^{-1}\tilde{\theta}(t)$.

D. We keep the CAC law in the loop until the moment of time $t_i \geq t_o + t_d$ when the switching inequality is violated. If the classical controller does not satisfy the inequality then, at $t = t_i$,

E. If the inequality $\triangle W_i(t) \leq 0$ is never violated, then there will not be any switching. **F.** If at some time, say t_i with $t_i \geq t_o + t_d$ and $t_i := t_o$, the controller that acting in the loop does not satisfy $\triangle W_i(t) \leq 0$, then another candidate will be put in the system. Note that there always exist a candidate that provides guaranteed minimum value of $\triangle W_i(t) \leq 0$ at that instant of time. Based on our above analysis, we now state the results as a state feedback design in the following Theorem 2.

Theorem 2: Consider the closed loop systems composed of (2) and (4) under the switching-logic defined in Algorithm 2. Then, there exists a time such that, according to Algorithm 2, the control law corresponding to the guaranteed decrease in the value of $\triangle W_i(t) \leq 0$ is tuned to the plant which ensures that all the tracking error signals in the closed loop systems are bounded.

We now consider that the velocity signals are not available and required to estimate the velocity signals by the output of the linear estimator (3) to formulate output feedback based adaptive output feedback controller (5) with $t_d > T_1(\varepsilon)$. We then present the main results in the following Theorem 3.

Theorem 3: Consider the closed-loop system (2), (3) and (5), under the switching-logic derived in Algorithm 2. Then, for any given $(e(0), \hat{e}(0)) \in \Omega_{co}$ and $\theta_i \in \Omega_i$ with $i \in \mathcal{M}$, there exists a small value of $\varepsilon > 0$ and $t_d > T_1(\varepsilon)$ such that the controller corresponding to a guaranteed decrease in the value of $\triangle W_i(t) \leq 0$ is tuned to the plant. Then, the states of the tracking error variables with multiple models based adaptive output feedback control laws are bounded.

Proof: The proof of Theorem 2 and Theorem 3 can be shown along the line of the logic introduced in Algorithm 2. The idea is to compare candidate controllers, $\tau^i(e, Q_d, \theta_i)$ with $i \in \mathcal{M}$, at each instant of time to see which candidate provides the highest decrease in the value of the following Lyapunov inequality $\triangle W_i(t) = V_i(t_s) - V_0(t)$ where $V_0(t) = \frac{1}{2}e^T(t)Q_{sm}e(t) + \frac{1}{2}\tilde{\theta}^T(t)\Gamma^{-1}\tilde{\theta}(t)$ and $V_i(t_s) = \frac{1}{2}e^T$ $(t_s)Q_{sm}e(t_s) + \frac{1}{2}\tilde{\theta}_i^T(t_s)\Gamma^{-1}\tilde{\theta}_i(t_s)$ with $\tilde{\theta}(t) = \left(\hat{\theta}(t) - \theta\right)$, $\tilde{\theta}_i(t_s) = (\theta_i(t_s) - \theta)$ and $t_s > t_o + t_d$ is the time when the parameter estimate, $\hat{\theta}(t)$, provided by classical adaptation mechanism is tuned to a model among finite set of candidate models $\theta_i(t_s)$ that best approximates the plant θ. This implies that the switching will occur only if $\triangle W_i(t)$ is a non-increasing sequence with respect to i as $\triangle W_i(t) \leq 0$. First, we simplify $\triangle W_i(t)$ as $\triangle W_i(t) = \frac{1}{2}e^T(t_s)Q_{sm}e(t_s) - \frac{1}{2}e^T(t)Q_{sm}e(t) - \frac{1}{2}\left[\Theta_i^T\Gamma^{-1}\Theta_i - 2\Theta_i^T\Gamma^{-1}(\theta_i(t_s) - \theta)\right]$ where $\Theta_i = \left(\theta_i(t_s) - \hat{\theta}(t)\right)$. To show $\triangle W_i(t) \leq 0$, we now calculate the value of $(\theta_i(t_s) - \theta)$. Let us define the torque prediction error model as $E_i(t) = Y(e, \dot{q}_d, \ddot{q}_d)(\theta_i - \theta)$ where $Y(e, \dot{q}_d, \ddot{q}_d)$ is the regressor matrix. We can write after some manipulations as $(\theta - \theta_i) = \xi^{-1}(t)\omega_i(t)$ where $\xi(t) > 0$ with $\xi(t) = \int_o^t Y(e, \dot{q}_d, \ddot{q}_d)Y^T(e, \dot{q}_d, \ddot{q}_d)d\tau, \omega_i(t) = \int_o^t Y(e, \dot{q}_d, \ddot{q}_d)E_i d\tau$ Since $e \in \Omega_c$, $Q_d \in \Omega_d$ and $\theta \in \Omega$, then $Y(e, \dot{q}_d, \ddot{q}_d)$ is bounded $\forall e \in \Omega_c$ and $\forall \theta \in \Omega$. Then, the control switching condition can be written as $\triangle W_i(t) = -\frac{1}{2}\Theta_i^T\Gamma^{-1}[\Theta_i + 2\xi^{-1}(t)\omega_i(t)] + \frac{1}{2}e^T(t_s)Q_{sm}e(t_s) - \frac{1}{2}e^T(t)Q_{sm}e(t) \leq 0$. This means that the parameter estimate $\hat{\theta}(t)$ will be reset into distributed models $\theta_i(t_s)$ only if the data has a strong evidence of the parameters [2]. If $\xi(t) = 0$, then there will not be any switching. If there is sudden change in the parameters, then the switching inequality will be hold. When the system is poorly excited $\xi(t) \approx 0$, then the error E_i needs to be very small to switch the parameter estimate obtained by standard adaptation law. During transient phase or large change in the parameter estimation errors, the data contains strong evidence of the parameters satisfying the switching inequality $\triangle W_i(t) \leq 0$ that improves the transient tracking performance. This implies that $\triangle W_i(t)$ is a non-increasing sequence and the switching will be occurred during transient phase or any large change in the parameter estimation errors. Then, we can conclude that $\Delta W_i(t)$ is bounded if the function $\Delta W_0(t)$ is bounded. This means that the Lyapunov function $V_i(t)$ is bounded if $V_0(t)$ is bounded. Notice from $\dot{V}_o(e, \tilde{\theta}) \leq -\lambda_{min}(\Pi)\|e\|^2$ that $V_0(t)$ is uniformly continuous and bounded. Then, we proceed the proof of Theorem 2 along the line of Algorithm 2.

Case 1: Let us first put (4) into the loop for a short time period $t \in [t_o, t_o + t_d]$. At $t \geq t_o + t_d$, we check $\triangle W_i(t) \leq 0$ using with standard adaptation mechanism. If it is satisfied, then we stay with that controller as long as $\triangle W_i(t) \leq 0$ holds. Then, we can conclude that the tracking errors are bounded.

Case 2: If the classical control law does not satisfy the switching criterion, then candidate controller will be put in the system based upon a guaranteed decrease in the value of $\triangle W_i(t)$.

Case 3: If the inequality $\triangle W_i(t) \leq 0$ is never violated, then there will not be any switching action.

Case 4: If there exists a time, say $t_i \geq t_i + t_d$ with $t_o + t_d = t_i$, such that the controller that acting in the loop does not satisfy $\triangle W_i(t) \leq 0$, then another candidate will be put via comparing a set of candidates which provides largest guaranteed minimum values of $\triangle W_i(t)$ at that instant of time. □

We now consider that the position and velocity signals in (4) are not measurable. The linear observer (3)

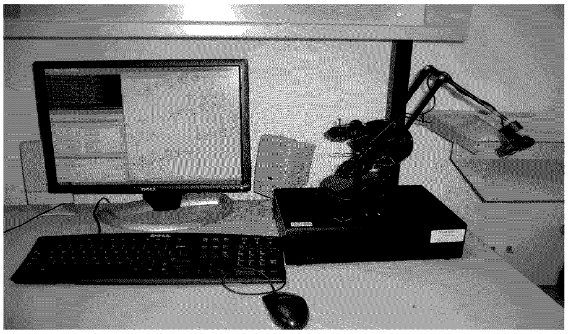


Fig. 1. Experimental setup with 3-DOF Phantom™ robotic system.

can be used to estimate unknown velocity signals to construct velocity independent adaptive feedback candidate controllers (5). The output feedback design (5) recovers the performance achieved under state feedback design (4). We consider small value of the observer design constant ε and replace $Y(\hat{e},\dot{q}_d,\ddot{q}_d)$ by $Y(e,\varepsilon\eta,\dot{q}_d,\ddot{q}_d)$. To ensure small bounded estimation error $\|\eta\|$, we choose $t_d > T_1(\varepsilon)$. Then, we follow the same steps of Algorithm 2 to proof Theorem 3.

3. EXPERIMENTAL RESULTS

In this section, we implement and evaluate the proposed multiple models based adaptive switching control strategy on 3-DOF robotic system. In our experiment, we use 3-DOF Phantom™ robotic system Premimum 1.5A robotic manipulators as shown in Fig. 1 provided by SensAble Technologies, Inc. The systems are equipped with standard gimbal end-effector. Each joint is attached with encoder for joint position measurement. For full state feedback design, the velocity signals are obtained by differentiating the joint position measurement as the system does not have velocity sensors. For output feedback design, the velocity signals are estimated by using a linear observer. We first obtain motion dynamics of Phantom™ robotic system as given in [54] The components of the motion dynamics for the given Phantom™ robot system are modeled as defined in [55]. The Phantom™ dynamical model depends on number of physical parameters. In our experimental evaluation, the parameter sets are defined as $\theta_1 = (I_{y1}+I_{y3}+I_{z2})$, $\theta_2 = \left(I_{y2}-I_{z2}+m_2r_2^2+m_3l_1^2\right)$, $\theta_3 = (I_{z3}-I_{x3}-I_{y3})$, $\theta_4 = (I_{x2}+m_2r_2^2+m_3l_1^2)$, $\theta_5 = \left(m_3r_3^2+I_{x3}\right)$, $\theta_6 = m_3r_3l_1$, $\theta_7 = g(m_2r_2+m_3l_1)$ and $\theta_8 = gm_3r_3$ where m_j with $j = 1,2,3$ are the mass of the j-th link, I_{xj}, I_{yj} and I_{zj} are the moment of inertia of each link and r_j are the center of masses for each link. In fact, one may choose different parameter sets which may provide different nonlinear regressor model $Y(q,\dot{q},\ddot{q})$ as used for our evaluation. Using above defined parameters, we then define nonlinear regressor model $Y(q,\dot{q},\ddot{q}) \in \Re^{3\times 8}$ [55]. To generate the desired reference trajectory for the robot to follow, a square wave with a period of 8 seconds and an amplitude of ± 0.5 radians is pre-filtered with a critically damped 2nd-order linear filter using a bandwidth of $\omega_n = 2.0$ rad./sec. The step reference inputs are not preferred as such initial jump may reduce the lifetime of the system. If one uses the step reference trajectory, then very small step sizes are required. We now examine the tracking performance of CE-based CAC law $\tau(e,Q_d,\hat{\theta}) = Y(e,\dot{q}_d,\ddot{q}_d)\hat{\theta} - K_P e_1 - K_D e_2$ and $\dot{\hat{\theta}} = -\Gamma Y^T(e,\dot{q}_d,\ddot{q}_d)S$ on the given system (6). To implement that, let us replace $\dot{q}$ in coriolis and centrifugal matrix $c(q,\dot{q})$ by $\dot{q}_r = (\dot{q}_2 - \lambda e_1)$ to construct the

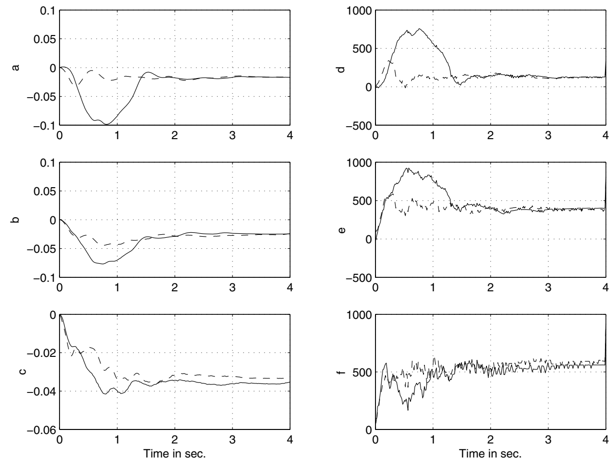


Fig. 2. a, b & c) The tracking errors (dash line is for Theorem 2 and solid-line is for CAC for joints 1, 2 and 3 in radians, d, e & f) The control inputs (dash line is for Theorem 2 and solid line is for CAC) for joints 1, 2 and 3 in Newton meter (Nm).

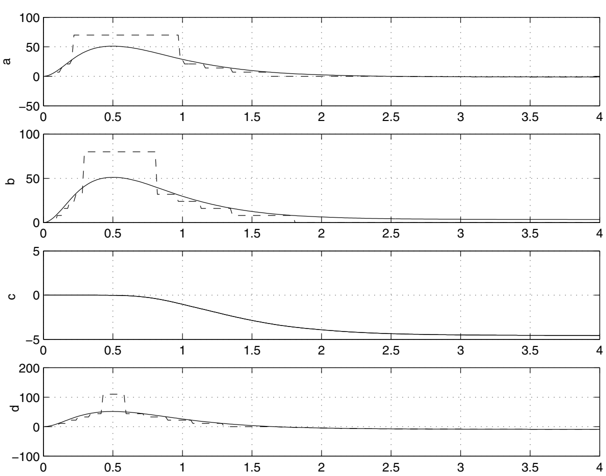


Fig. 3. a, b, c & d) The parameter estimate θ_1, θ_2, θ_3 and θ_4 with CAC algorithm (solid line) and distributed candidate models of Theorem 2 (dash line) in kgm^2.

regressor model $Y(e, \dot{q}_d, \ddot{q}_d)$. The values of PD controller design parameters are selected as $\lambda_0 = 2$, $K_P = diag(750, 1200, 1200)$ and $K_D = diag(500, 800, 800)$. The values of Γ are chosen as $\Gamma = 500 I_{8\times 8}$ via using trial and error search technique. With these parameters, we then apply CAC algorithm on Phantom$^{\mathbf{TM}}$ system to track given desired trajectory. The tested results are given in Figs. 2 to 4 (solid line). Let us experimentally compare multiple parameter models based adaptive switching control (4) with the CAC design on the same system. In our experiment, we assume that the parameters $\theta_1, \theta_2, \theta_3, \ldots, \theta_8$, for Phantom$^{\mathbf{TM}}$ systems are arbitrarily distributed into a finite number of smaller compact subsets Ω_1 to Ω_8 as $\Omega_1 = \bigcup_{i=1}^{11}\{\Omega_{1i}\} = \{-70, -63, \ldots, 63, 70\}$, $\Omega_2 = \bigcup_{i=1}^{11}\{\Omega_{2i}\} = \{-80, -72, \ldots, 72, 80\}$, $\Omega_3 = \bigcup_{i=1}^{11}\{\Omega_{3i}\} = \{-60, -54, \ldots, 54, 60\}$, $\Omega_4 = \bigcup_{i=1}^{11}\{\Omega_{4i}\} = \{-120, -108, \ldots, 108, 120\}$, $\Omega_5 = \bigcup_{i=1}^{11}\{\Omega_{5i}\} = \{-20, -18, \ldots, 18, 20\}$, $\Omega_6 = \bigcup_{i=1}^{11}\{\Omega_{6i}\} = \{-20, -18, \ldots, 18, 20\}$, $\Omega_7 = \bigcup_{i=1}^{11}\{\Omega_{7i}\} = \{-320, -288, \ldots, 288, 320\}$, $\Omega_8 = \bigcup_{i=1}^{11}\{\Omega_{8i}\} = \{-320, -288, \ldots, 288, 320\}$. As we can see from parameter distributions that the upper bounds of the parameters and the number of distributed candidate models are arbitrarily chosen to investigate overall tracking convergence property with respect to unknown parameter bounds, smaller number of the distributed models and nonuniform parameter distributions. We can also see from identified and distributed candidate models that none of the distributed candidates are exactly equal to the identified parameters. We then design distributed candidate controller for each of the distributed models and implement Theorem 1 on Phantom$^{\mathbf{TM}}$ system. The controller design parameters are chosen as $\lambda_0 = 2$, $K_P = diag(750, 1200, 1200)$ and $K_D = diag(500, 800, 800)$. The small value of t_d is chosen arbitrarily as $t_d = 0.003$. The tested results are given in Figs. 2 to 4 (dash line). By comparing solid and dash line of Fig. 2, we can notice that the transient tracking errors and control efforts under Theorem 2 are smaller than the tracking errors and control inputs obtained with CAC algorithm. We now implement CE based adaptive output feedback control algorithm $\tau(\hat{e}, Q_d, \hat{\theta}) = Y(\hat{e}, \dot{q}_d, \ddot{q}_d)\hat{\theta} - K_P\hat{e}_1 - K_D\hat{e}_2$ where $\dot{\hat{q}}_r = (\hat{e}_2 + \dot{q}_d - \lambda\hat{e}_1)$ and $\hat{e}$ is replaced by observer (3) on the given system. We use the same controller design parameters that used in our previous experiment as $\lambda_0 = 2$, $K_P = diag(750, 1200, 1200)$, $K_D = diag(500, 800, 800)$ and $\Gamma =_{8\times 8}$. The observer design parameters are chosen as $H_1 = 15 I_{3\times 3}$ and $H_2 = 5 I_{3\times 3}$ and $\varepsilon = 0.05$. The tested results are depicted in Figs. 5 to 7 (solid line). To reduce the tracking errors further, we reduce the value of the observer design constant ε (that is increase the observer gains) from $\varepsilon = 0.05$ to $\varepsilon = 0.04$. However, due to joint oscillation and control chattering activity, the control system goes unbounded as soon as we start the experiment. Finally, we implement adaptive switching control algorithm introduced in Theorem 3 on the given

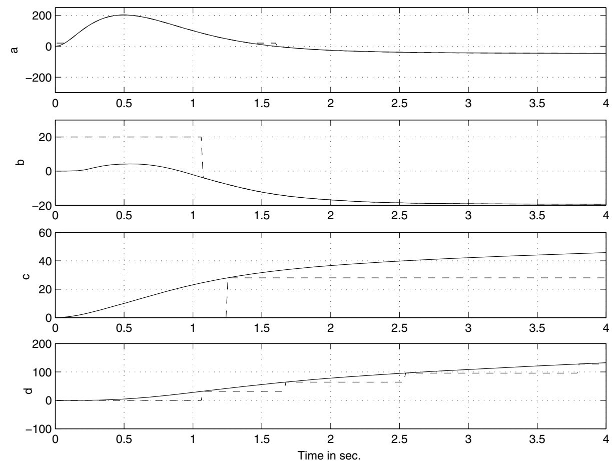


Fig. 4. a, b, c & d) The parameter estimate θ_5, θ_6, θ_7 and θ_8 under CAC algorithm (solid line) and distributed adaptive model of Theorem 2 (dash line) in kgm^2.

Phantom$^{\mathbf{TM}}$ system. The controller-observer design parameters are kept similar to our previous experiment for CAOFB design as $\lambda_0 = 2$, $K_P = diag(750, 1200, 1200)$, $K_D = diag(500, 800, 800)$, $H_1 = 15 I_{3\times 3}$, $H_2 = 5 I_{3\times 3}$ and $\varepsilon = 0.05$. The small values of t_d, k_{f1} and k_{f2} are chosen as $t_d = 0.005$, $k_{f1} = 0$ and $k_{f2} = 0$, respectively. Notice that we select the values of k_{f1} and k_{f2} are zero instead constants in order to investigate the tracking property of Theorem 3 with respect to vanishing estimation errors. We then follow the steps of Algorithm 2 to implement Theorem 3. The tested results are depicted in Figs. 5 to 7 (dash line). From the solid and dash line of the Fig. 5, we can notice that the tracking errors and control efforts under multiple models based adaptive switching control design are smaller than the tracking errors and control inputs obtained with the single model based CAC approach. This means that the proposed method saves power consumption by reducing actuator input. We can also see from Fig. 5 that the joint oscillation under Theorem 3 reduces significantly. The sampling time was set to 0.005 sec. for all the experimental results presented in this section.

4. CONCLUSION

In this paper, we introduced multiple parameter models based adaptive switching control for a class of nonlinear mechanical systems. The proposed method can be used to improve the transient tracking performance in the presence of large scale parametric uncertainties. The design reduced the control gains/control efforts from the single model based adaptive control paradigm through on-line estimation of the Lyapunov function equality. The proposed design increased the convergence speed of adaptation mechanism via tuning the parameter/control from CAC algorithm into a finite set of dis-

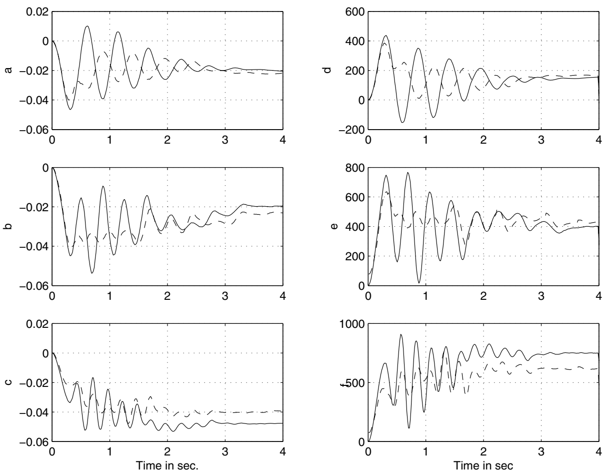


Fig. 5. a, b & c) The tracking errors (dash line is for Theorem 3 and solid-line is for CAC) for joints 1, 2 and 3 in radians,d, e & f) The control inputs (dash line is for Theorem 3 and solid line is for CAC) for joints 1, 2 and 3 in Nm.

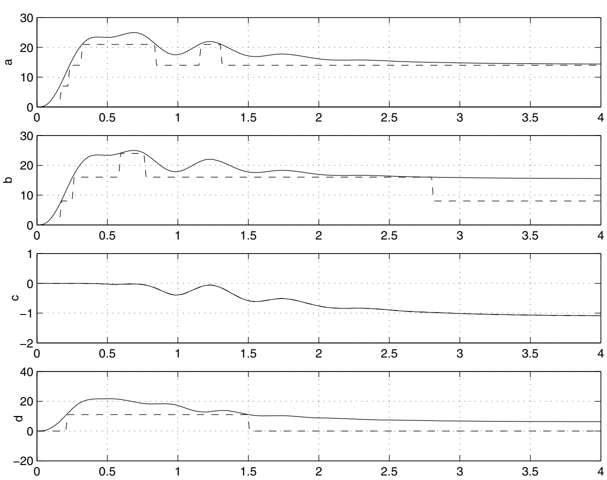


Fig. 6. a, b, c & d) The parameter estimate θ_1, θ_2, θ_3 and θ_4 with CAC algorithm (solid line) and Theorem 3 (dash line) in kgm^2.

tributed model/control that best approximates the plant at each instant of time. Experimental results on a 3-DOF Phantom$^{\mathbf{TM}}$ robot manipulator have been used to demonstrate the effectiveness of the proposed design for real-time applications. Notice from our evaluation that the proposed design requires smaller observer-controller gains than classical CE based CAC adaptive design to guarantee stability and tracking performance in the presence of large-scale modeling errors and disturbance uncertainties.

It is also noticed from our evaluation results that multiple model based hybrid design requires smaller control efforts than the single model based adaptive design resulting in smaller energy consumption of the robotic manipulator. As a result, the proposed multiple model based hybrid design can reduce the energy consumption of the industrial robotic manipulator for large scale industrial automation.

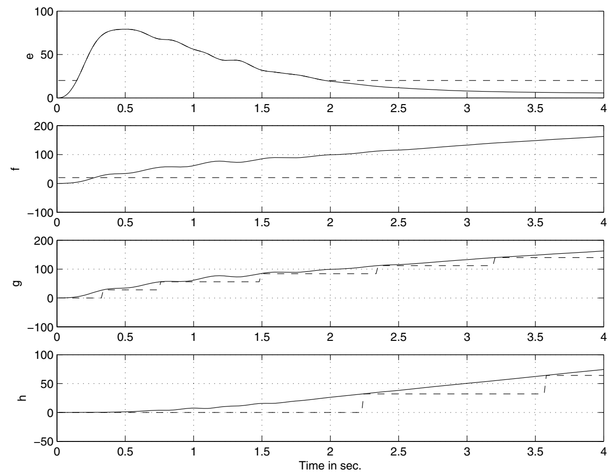


Fig. 7. a, b, c & d) The parameter estimate θ_5, θ_6, θ_7 and θ_8 under CE-based CAC algorithm (solid line) and Theorem 3 (dash line) in kgm^2.

REFERENCES

[1] H. Asada and J. J. E. Slotine, *Robot Analysis and Control*, Wiley, New York, 1986.

[2] K. J. Astrom and B. Wittenmark, *Adaptive Control*, *Wesley*, New York, 1995.

[3] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*, Prentice Hall, New Jersy, 1988.

[4] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice Hall, Upper Saddle River, NJ, 1991.

[5] H. Berghuis, R. Ortega, and H. Nijmeijer, “A robust adaptive robot controller,” *IEEE Transactions on Robotics and Automation*, vol. 9, pp. 825-830, 1993. [click]

[6] C. C. Cheah, C. Liu, and J. J. E. Slotine, “Adaptive tracking control for robots with unknown kinematic and dynamic properties,” *IEEE Int. J. Robot. Res.*, vol. 25, pp. 283-296, 2006. [click]

[7] C. C. Cheah, M. Hirano, S. Kawamura, and S. Arimoto, “Approximate Jacobian control for robots with uncertain kinematics and dynamics,” *IEEE IEEE Trans. Robot. Autom.*, vol. 19, pp. 692-702, 2003.

[8] M. Erlic and W. S. Lu, “A reduced-order adaptive velocity observer for manipulator control,” *IEEE Transaction on Robotics and Automation*, vol. 11, pp. 293-303, 1995. [click]

[9] P. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice Hall, 1996.

[10] W. Gueaieb, F. Karray, and S. Al Sharhan, “A robust hybrid intelligent position/force control scheme for cooperative manipulators,” *IEEE IEEE/ASME Trans. Mechatronics*, vol. 12, pp. 109-125, 2007. [click]

[11] S.-H. Hsu and L.-C. Fu, "Adaptive decentralized control of robot manipulators driven by current-fed induction motors," *IEEE/ASME TTransactions on Mechatronics*, vol. 13, pp. 576-586, 2008.

[12] V. Lertpiriyasuwat and M. C. Berg, "Adaptive real-time estimation of end-effector position and orientation using precise measurements of end-effector position," *IEEE/ASME Transactions on Mechatronics*, vol. 11, pp. 304-319, 2006. [click]

[13] S. Lim, D. Dawson, and K. Anderson, "Re-examining the Nicosia-Tomei robot observer-controller from a backstepping perspective," *IEEE Transaction on Control Systems Technology*, vol. 4, pp. 304-310, 1996. [click]

[14] W. S. Lu and M. Q. H. Meng, "Regressor formulation of robot dynamics: computation and applications," *IEEE Transactions on Robotics and Automation*, vol. 9, pp. 323-333, 1993. [click]

[15] B. Martensson, "The order of any stabilizing regulator is sufficient a priori informaton for adaptive stabilization," *System and Control letter*, vol. 6, no. 2, pp. 87-91, 1985.

[16] M. Q. H. Meng and W. S. Lu, "A unified approach to stable adaptive force/position control of robot manipulators," *Proc. of the American Control Conference*, 1994.

[17] M. W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, Wiley, New York, 1989.

[18] B. Dexu, S. Wei, Y. Hongshan, W. Cong, and Z. Hui, "Adaptive robust control based on RBF neural networks for duct cleaning robot," *International Journal of Control, Automation, and Systems*, vol. 13, no. 2, pp. 475-487, 2015. [click]

[19] R. H. Middleton, G. C. Goodwin, D. J. Hill, and D. Q. Mayne, "Design issues in adaptive control," *IEEE Transaction on Automatic Control*, vol. 33, no. 1, pp. 50-58, 1988. [click]

[20] J. P. Hespanha, D. Liberzon, and A. S. Morse, "Hysteresis-based switching algorithms for supervisory control of uncertain systems," *Automatca*, vol. 39, pp. 263-272, 2003. [click]

[21] A. Morse, D. Q. Mayne, and G. C. Goodwin, "Application of hystersisi switching in parameter adaptive control," *IEEE Transaction on Automatic Control*, vol. 37, no. 9, pp. 1343-1354, 1992. [click]

[22] A. S. Morse, "Control using logic-based switching," *In Trends in Control: An European Perspective*, A. Isidori, Ed., Springer, London, U.K., pp. 69-113, 1995.

[23] A. S. Morse, "Supervisory control of families of linear set-point controller, part I: robustness," *IEEE Transaction on Automatic Control*, vol. 41, pp. 1413-1431, 1996. [click]

[24] A. S. Morse, "Supervisory control of families of linear set-point controller, part I: Exact matching," *IEEE Transaction on Automatic Control*, vol. 42, no. 1, pp. 1500-1515, 1997. [click]

[25] J. P. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," *Proc. of 39th Conference on Decision and Control*, pp. 2655-2660, 1999.

[26] J. P. Hespanha and A. S. Morse, "Certainty equivalence implies detectability," *System and Control Letter*, vol. 36, pp. 1-13, 1999. [click]

[27] R. A. Hilhorst, J. V. Amerongen, P. Lohnberg, and H. J. A. F. Tulleken, "A supervisor for control of mode-switch processes," *Automatca*, vol. 30, pp. 1319-1331, 1994. [click]

[28] S. J. Cusumano and K. Polla, "Adaptive control of uncertain systems: A new approach," *Proc. American Automatic Control Conference*, pp. 355-359, 1988.

[29] M. Fu and B. R. Barmish, "Adaptive stabilization of linear systems via switching control," *IEEE Transaction on Automatic Control*, pp. 1079-1103, 1986.

[30] B. Martensson, "The order of any stabilizing regulator is sufficient a priori informaton for adaptive stabilization," *System and Control Letter*, vol. 6, no. 2, pp. 87-91, 1985.

[31] D. E. Miller, *Adaptive Control of Uncertain Systems*, Ph.D. Dissertation, University of Toronto, 1989.

[32] D. Borrelli, A. S. Morse, and E. Mosca, "Discrete-time supervisory contorl of families of two-degrees-of-freedom liner set-point controllers," *IEEE Transaction on Automatic Control*, vol. 44, no. 11, pp. 178-181, 1999. [click]

[33] K. S. Kumpati and J. Balakrishnan, "Improving transient response of adaptive control systems using multiple models and switching," *Proc. of the 32nd Conference on Decision and Control*, San Antonio, Texas, pp. 1067-10723, 1993.

[34] K. S. Kumpati and J. Balakrishnan, "Improving transient response of adaptive control systems using multiple models and switching", *IEEE Transaction on Automatic Control*, vol. 39, no. 9, pp. 1861-1866, 1994. [click]

[35] K. S. Kumpati and J. Balakrishnan, "Adaptive control using multiple models," *IEEE Transaction on Automatic Control*, vol. 42, no. 2, pp. 171-187, 1997. [click]

[36] G. Battistelli, E. Mosca, M. G. Safonov, and P. Tesi, "Stability of unfalsified adaptive switching control in noisy environments," *IEEE Transactions on Automatic Control*, vol. 46, pp. 249-259, 2010.

[37] M. G. Safonov and T. C. Tsao, "The unfalsified control concept and learning," *IEEE Transaction on Automatic Control*, vol. 42, pp. 843-847, 1997. [click]

[38] M. Stefanovic and M. G. Safonov, "Safe adaptive switching control: Stability and convergence," *IEEE Trans. on Automatic Control*, vol. 53, pp. 2012-2021, 2011. [click]

[39] R. Wang, A. Paul, M. Stefanovic, and M. G. Safonov, "Cost detectability and stability of adaptive control systems," *Int. J. Robust Nonlinear Control*, vol. 17, pp. 549-561, 2005.

[40] D. Angeli and E. Mosca, "Lyapunov-based switching supervisory control of nonlinear uncertain systems," *IEEE Transaction on Automatic Control*, vol. 47, no. 3, pp. 500-505, 2002.

[41] D. Angeli and E. Mosca, "Lyapunov-based switching supervisory control of nonlinear uncertain systems," *Proc. of the American Control Conference*, Arlington, VA, USA, June 25-27, pp. 967-972, 2001.

[42] D. Angeli and E. Mosca, "Adaptive switching supervisory control of nonlinear systems with no prior knowledge of noise bounds," *Proc. of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada, USA, pp. 1187-1192, 2002.

[43] S. S. Baldi, G. Battistelli, E. Mosca, and P. Tesi, "Multi-model unfalsified adaptive switching supervisory control," *Automatica*, vol. 46, pp. 249-259, 2010. [click]

[44] S. Baldi, G. Battistelli, E. Mosca, and P. Tesi, "Multi-model unfalsified adaptive switching control: Test functionals for stability and performance," *International Journal of Adaptive Control and Signal Processing*, vol. 25, pp. 593-612, 2011. [click]

[45] S. Baldi, G. Battistelli, D. Mari, E. Mosca, and P. Tesi," Multiple-model adaptive switching control for uncertain multivariable systems," *Proc. of 50th IEEE CDC-ECC*, Orlando, FL, USA, December 12-15, pp. 6153-6158, 2011. [click]

[46] G. Battistelli, E. Mosca, and P. Tesi, "Dealing with plant variations in multi-model unfalsified switching control via adaptive memory selection," *Proc. of 50th IEEE CDC-ECC*, Orlando, FL, USA, pp. 361-371, December 12-15 2011.

[47] G. Battistelli, J. P. Hespanha, E. Mosca, and P. Tesi, "Model-free adaptive switching control of time-varying plants," *IEEE Trans. on Automatic Control*, vol. 58, no. 5, pp. 1208-1220, 2013. [click]

[48] C. Hua and P. X. Liu, "Convergence analysis of teleoperation systems with unsymmetric time-varying delays," *IEEE Transaction on Circuits and Systems-II*, vol. 56, pp. 240-244, 2005. [click]

[49] H. K. Khalil, "Adaptive output feedback control of nonlinear systems represented by input-output models," *IEEE Trans. on Automatic Control*, pp. 177-188, 1996. [click]

[50] J.-B. Pomet and L. Praly, "Adaptive nonlinear regulation estimation from the Lyapunov equation," *IEEE Transaction on Automatic Control*, vol. 37, pp. 729-740, 1992. [click]

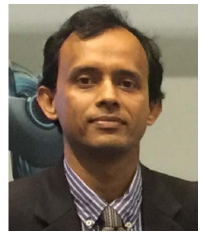
[51] H. Schwartz and S. Islam, "An evaluation of adaptive robot control via velocity estimated feedback," *Proc. on Control and Applications*, Montreal, Quebec, May 30-June 1, pp. 125-133, 2007.

[52] S. Islam and P. X. Liu, "Adaptive output feedback control for robot manipulators using Lyapunov-based switching," *Proc. IEEE/RSJ International Conference on IROS*, St. Louis, USA, pp. 899-905, October 11-15 2009.

[53] S. Islam and P. X. Liu, "A hybrid adaptive control approach for robust trajectory tracking of robotic manipulators: Theory and experiment," *Robotica*, vol. 29, no. 2, pp. 255-269, 2010.

[54] S. Islam, *Lyapunov-based Hybrid Control for Robust Trajectory Tracking of Robotic Manipulators*, PhD Thesis, Carleton University, Ottawa, Ontario, Canada, 2010.

[55] I. G. Polouchine, *Force-Reflecting Teleoperation Over Wide-area Networks*, PhD Thesis, Carleton University, Ottawa, Canada, 2008.



Shafiqul Islam obtained his PhD degree in Electrical and Computer Engineering from Ottawa-Carleton Institute for Electrical and Computer Engineering at Carleton University, Canada. Dr. Islam has a master in Control Engineering and B. Sc. in Electrical and Electronic Engineering. His research funded by many organizations including Natural Sciences and Engineering Research Council of Canada (NSERC) Canada, CMC Electronics, Harvard University, USA, Carleton University, Canada, Lakehead University, Canada, Ministry of Education, Culture, Sports, Science and Technology of Japan, etc. He was awarded Research Excellence in Science and Engineering from Carleton University, Canada for his outstanding contribution to research and development. Dr. Islam awarded prestigious NSERC Canada Visiting Research Fellowship award for visiting national and international research laboratory. He is the author and co-author of more than 70 peer-reviewed journal and conference articles in the area of Robotics and Automation, Power System Stability and Control and Smart Power Grid Networks and Control.

Peter X. Liu received his Ph.D. degree from the University of Alberta in 2002. He has been with the Department of Systems and Computer Engineering, Carleton University, Canada since July 2002 and is currently a Professor and Canada Research Chair. Dr. Liu has published more than 300 research articles and serves as an Associate Editor for several journals including IEEE/ASME Transactions on Mechatronics and IEEE Transactions on Automation Science and Engineering. He received a 2007 Carleton Research Achievement Award, a 2006 Province of Ontario Early Researcher Award, a 2006 Carty Research Fellowship, the Best Conference Paper Award of the 2006 IEEE International Conference on Mechatronics and Automation, and a 2003 Province of Ontario Distinguished Researcher Award. Dr. Liu is a licensed member of the Professional Engineers of Ontario (P.Eng) and a senior member of IEEE.

Jorge Dias has an 'Agregacao' (Habilitation) degree and a Ph.D. on Electrical Engineering by the University of Coimbra, specialization in Control and Instrumentation. Jorge Dias have been Associated Professor at the University of Coimbra with activities in the Department of Electrical Engineering and Computers(www.deec.uc.pt) and the Institute of Systems and Robotics (ISR) (www.isr.uc.pt) from the University of Coimbra (UC) (www.uc.pt). Jorge Dias coordinated the Mobile Robotics Laboratory from Instituto of Systems and Robotics and the Laboratory of Systems and Automation (LAS) (http://las.ipn.pt) from the Instituto Pedro Nunes (IPN) (www.ipn.pt). Since July 2011, Jorge Dias is acting as Associate Professor from ECE/Robotics at Khalifa University (UAE) at Abu Dhabi. Dr. Dias has published more than 300 research articles in the area of robotics.

Lakmal D. Seneviratne earned his PhD in Mechanical Engineering from King's College London (KCL). His research interests are centered on robotics and automation, with particular emphasis on increasing the autonomy of robotic systems interacting with complex dynamic environments. He has published over 300 peer reviewed articles on these topics. Before joining Khalifa University he was a Professor of Mechatronics at Kings College London. He was the founding Director of the Centre for Robotics at KCL (1994-2010) and the Head of Division of Engineering at KCL (2004-2009). Lakmal Seneviratne has been appointed as Associate Provost for Research and Graduate Studies of Khalifa University of Science, Technology and Research (KUSTAR).