

# Adaptive Output Feedback Control for Miniature Unmanned Aerial Vehicle

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**Abstract**—In this paper, we address output feedback control problem for miniature unmanned aerial vehicle (MUAV). We first design state feedback based nonlinear control by assuming that states are available for feedback. Then, we use observer to estimate the velocity signals to develop output feedback based nonlinear controller for MUAV. The convergence condition with output feedback design is given by Lyapunov method. The analysis shows that the performance achieved under state feedback can be recovered by using output feedback as the values of observer gain converges to zero. Evaluation results are presented to demonstrate the validity of the theoretical argument of the proposed design for real-time applications.

## I. INTRODUCTION

Over the past decade, various autonomous flight tracking system have been proposed for MUAV in the literature. These designs can be divided into two categories as classical and nonlinear flight control system. Classical PID, LQR and other model based designs can be found in [1, 4, 5, 8, 10, 13, 23, 19, 25]. Since the classical designs cannot cope with uncertainty, nonlinear control technique has also been used for developing autonomous flight tracking system for MUAV [2, 3, 6, 9, 11, 12, 13, 15, 16, 17, 18, 20, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, and many others]. These designs can be used to deal with stability and tracking control problem of MUAV against bounded uncertainty. However, most reported designs are required to have the full state measurement of the linear and rotational dynamics which may not be realistic in real-time applications. On the other hand, measurements may be corrupted with severe noise restricting practical applications due to the presence of chattering phenomenon. To deal with this problem, authors in [21] proposed observer based control technique for MUAV. The design was motivated based on the method developed in [23, 24]. The drawback of such observer can be found in [22].

In this paper, we propose output feedback based nonlinear control strategy for MUAV system. The design has two steps. In the first step, the states are assumed to be available for feedback and design state feedback based nonlinear control for MUAV system. Then, observer is used to estimate unknown velocity states and design output feedback based control for MUAV. Lyapunov method is used to design and analyze the convergence property of the closed loop dynamics. For output feedback design, separation principle

is used to analyze the singularly perturbed slower and faster subsystem. It is shown in our analysis that the performance achieved with state feedback design can be recovered by using output feedback design provided that the observer dynamics are faster than the closed loop system dynamics. Evaluation results on quadrotor UAV are given to validate the effectiveness of the proposed design for real-time applications.

This paper is organized as follows. The dynamical model of the MUAV is given in section II. Section II also develops output feedback based nonlinear control algorithm. Evaluation results are given in section III. Finally, conclusion is given in section IV.

## II. DYNAMICAL MODEL, ALGORITHM DESIGN AND STABILITY ANALYSIS

Before developing output feedback based nonlinear control algorithm, let us introduce dynamical model of the quadrotor UAV system. The detail MUAV model can be found in [12, 13, 35, 36, 38]. The motion dynamic of quadrotor UAV system can be written as

$$\ddot{\mathcal{K}} = \beta U_L - \mathcal{C}\dot{\mathcal{K}} - \delta_l \quad (1)$$

$$\ddot{\Theta} = \mathcal{M}U - \eta \mathcal{D}(\Theta, \dot{\Theta}) - \xi \dot{\Theta} - \mathcal{B}\dot{\Theta} \times \mathcal{B}I\dot{\Theta} - \mathcal{B}\dot{\Theta} \times \Sigma_{i=1}^4 I_r \omega_i \quad (2)$$

where  $\mathcal{M} = (IB)^{-1}$ ,  $\eta = \mathcal{B}^{-1}$ ,  $\beta = (m\mathcal{R}_t^T)^{-1}$ ,  $m$  denotes the mass of the vehicle,  $\mathcal{C} = m^{-1}L$ ,  $L = \text{diag}[L_{d1}, L_{d2}, L_{d3}]$ ,  $\delta_l = Ng$ ,  $N = [0, 0, 1]^T$ ,  $g = 9.81 \frac{m}{s^2}$ ,  $I_r$  is the inertia of the rotor,  $\omega_i$  are the angular velocities of the rotor,  $\xi = I^{-1}M$ ,  $I = \text{diag}[I_x, I_y, I_z]$ ,  $M = \text{diag}[M_1, M_2, M_3]$ ,  $\Theta(t) = [\phi(t) \theta(t) \varphi(t)]^T$  represents the attitude of the vehicle,  $\mathcal{K}(t) = [x(t) y(t) z(t)]^T$  denotes the position of the vehicle,  $V(t) = [V_1(t) V_2(t) V_3(t)]^T$  defines translational velocity and  $\Omega(t) = [\Omega_1(t) \Omega_2(t) \Omega_3(t)]^T$  represents rotational velocity,

$$\mathcal{R}_t = \begin{bmatrix} C_\phi C_\varphi & S_\phi S_\theta C_\varphi - C_\phi S_\varphi & C_\phi S_\theta C_\varphi + S_\phi S_\varphi \\ C_\theta S_\varphi & S_\theta S_\phi S_\varphi + C_\theta C_\varphi & C_\theta S_\phi C_\varphi - S_\theta S_\varphi \\ -S_\phi & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\phi C_\theta \end{bmatrix} \quad (3)$$

with  $\sin(\cdot)$  and  $\cos(\cdot)$  represented by  $S_{(\cdot)}$  and  $C_{(\cdot)}$ , respectively.

Let us first assume that the states of the linear and angular dynamic are measurable. Then, we design state feedback based flight tracking control system. The goal is to develop

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state feedback controller such that the actual linear and angular states converge to the desired linear and angular states. The linear and angular dynamical error model can be presented by the following state space equation

$$\dot{e}_1 = e_2, \dot{e}_2 = -\beta U_L + \delta_l + \mathcal{C}x_2 - \mathcal{H}_a + \ddot{x}_{1d} \quad (4)$$

$$\dot{e}_3 = e_4, \dot{e}_4 = -\mathcal{M}U - \Pi_t + \ddot{x}_{3d} \quad (5)$$

where  $\Pi_t = \mathcal{A}(x_3, x_4) + \mathcal{H}_b$ ,  $\mathcal{A}(x_3, x_4) = -\eta \mathcal{D}(x_3, x_4) - \xi x_4 - \mathcal{B}x_4 \times \mathcal{B}I x_4 - \mathcal{B}x_4 \Sigma_{i=1}^4 I_r \omega_i$ ,  $x_1 = \mathcal{K}$ ,  $x_2 = \dot{\mathcal{K}}$ ,  $x_3 = \Theta$ ,  $x_4 = \dot{\Theta}$ ,  $x_{1d} = \mathcal{K}_d$ ,  $x_{2d} = \dot{\mathcal{K}}_d$ ,  $x_{3d} = \Theta_d$ ,  $x_{4d} = \dot{\Theta}_d$  with  $\mathcal{K}_d = [x_d, y_d, z_d]^T$ ,  $\Theta_d = [\phi_d, \theta_d, \psi_d]^T$ ,  $e_1 = (x_{1d} - x_1)$ ,  $e_2 = (\dot{x}_{1d} - \dot{x}_1)$ ,  $e_3 = (x_{3d} - x_3)$ ,  $e_4 = (\dot{x}_{3d} - \dot{x}_3)$ .

Then, we propose the following state feedback control algorithm for linear dynamic

$$U_L = \beta^{-1}(\ddot{x}_{1d} + k_d x_2 + \lambda_L e_2 + \delta_l + k_L S_L - u_l) \quad (6)$$

$$u_l = \hat{\theta}_1 \text{sign}(S_L), \dot{\hat{\theta}}_1 = -\Gamma_1 \text{sign}^T(S_L) S_L \quad (7)$$

where  $\tilde{\theta}_1 = (\theta_1 - \hat{\theta}_1)$ ,  $k_d = \text{diag}[k_{d1}, k_{d2}, k_{d3}]$ ,  $\lambda_L = \text{diag}[\lambda_{Lx}, \lambda_{Ly}, \lambda_{Lz}]$ ,  $k_L = \text{diag}[k_{L1}, k_{L2}, k_{L3}]$ ,  $U_L = [U_{lx}, U_{ly}, U_{lz}]^T$ ,  $\text{sign}(S_L) = \text{diag}[\text{sign}(S_{Lx}), \text{sign}(S_{Ly}), \text{sign}(S_{Lz})]$ ,  $\text{sign}(\cdot)$  is the sign function,  $\hat{\theta}_1$  is the estimate of  $\theta_1$ ,  $\Gamma_1 = \text{diag}[\Gamma_{1a}, \Gamma_{1b}, \Gamma_{1c}]$  with  $\Gamma_{1i} > 0$  and  $i = (a, b, c)$ ,  $S_L = (e_2 + \lambda_L e_1)$  with positive definite constant diagonal matrices  $\lambda_L$ . It is assumed that desired tasks  $\mathcal{K}_d$  and their first, second and third derivatives are assumed to be bounded. The term  $\mathcal{H}_a$  is also assumed to be bounded as  $\|\mathcal{H}_a\| \leq \theta_1$  with  $\theta_1 = [\theta_{1a}, \theta_{1b}, \theta_{1c}]^T$ .

We then introduce the following attitude control algorithm for rotational dynamic

$$U = \mathcal{M}^{-1}(\ddot{x}_{3d} + \lambda_A e_4 + k_A S_A - u_a) \quad (8)$$

$$u_a = \hat{\theta}_2 \text{sign}(S_A), \dot{\hat{\theta}}_2 = -\Gamma_2 \text{sign}^T(S_A) S_A \quad (9)$$

where  $\tilde{\theta}_2 = (\theta_2 - \hat{\theta}_2)$ ,  $\lambda_A = \text{diag}[\lambda_{A1}, \lambda_{A2}, \lambda_{A3}]$ ,  $k_A = \text{diag}[k_{A1}, k_{A2}, k_{A3}]$ ,  $\Gamma_2 = \text{diag}[\Gamma_{2a}, \Gamma_{2b}, \Gamma_{2c}]$  with  $\Gamma_{2i} > 0$  and  $i = (a, b, c)$ ,  $U = [U_2, U_3, U_4]^T$ ,  $\text{sign}(S_A) = \text{diag}[\text{sign}(S_{A\phi}), \text{sign}(S_{A\theta}), \text{sign}(S_{A\psi})]$ ,  $\hat{\theta}_2$  is the estimate of  $\theta_2$ ,  $S_A = (e_4 + \lambda_A e_3)$  with positive definite constant diagonal matrices  $\lambda_A$ . The design is assumed that desired tasks  $\Theta_d$  and their first, second and third derivatives are assumed to be bounded. The term  $\Pi_t$  is also assumed to be bounded as  $\|\Pi_t\| \leq \theta_2$  with  $\theta_2 = [\theta_{2a}, \theta_{2b}, \theta_{2c}]^T$  and  $\|\mathcal{H}_b\| \leq \theta_o$ . Then, we can state the following Theorem 1.

**Theorem 1:** *There exists positive definite constant diagonal matrices  $k_A$ ,  $k_L$ ,  $k_d$ ,  $\lambda_A$ ,  $\lambda_L$ ,  $\Gamma_1$  and  $\Gamma_2$  such that the linear and angular error states in the closed loop systems formulated by equation (4)-(9) are bounded and their bounds asymptotically converge to zero.*

Note that the result presented in above Theorem assumes that full states measurement of the linear and angular dynamic are available for feedback. However, full state measurements of the vehicle dynamic may not be available or difficult to realize for real-time implementation. In this work, we propose to use observer to estimate unknown velocity signals of the linear and angular dynamics and develop output feedback based nonlinear flight tracking control system for

MUAV. To develop that, we define  $y_1 = \mathcal{K}$ ,  $y_2 = \dot{\mathcal{K}}$ ,  $y_3 = \Theta$  and  $y_4 = \dot{\Theta}$ . Then, we can write the dynamical model (1), (2) in state space form as

$$\begin{aligned} \dot{y}_1 &= y_2, \dot{y}_2 = \mathcal{W}_1(y_2, U_L) \\ \dot{y}_3 &= y_4, \dot{y}_4 = \mathcal{W}_2(y_3, y_4, U) \end{aligned} \quad (10)$$

with  $\mathcal{W}_1(y_2, U_L) = \beta U_L - \mathcal{C}y_2 - \gamma - \mathcal{H}_a$  and  $\mathcal{W}_2(y_3, y_4, U) = \mathcal{M}U + \Pi_t$ . The following observer is used to estimate the linear and angular velocity of the vehicle dynamic

$$\begin{aligned} \dot{\hat{y}}_1 &= \hat{y}_2 + \frac{H_1}{\epsilon_l}(y_1 - \hat{y}_1), \dot{\hat{y}}_2 = \frac{H_2}{\epsilon_l^2}(y_1 - \hat{y}_1) \\ \dot{\hat{y}}_3 &= \hat{y}_4 + \frac{H_3}{\epsilon_a}(y_3 - \hat{y}_3), \dot{\hat{y}}_4 = \frac{H_4}{\epsilon_a^2}(y_3 - \hat{y}_3) \end{aligned} \quad (11)$$

with small constant  $\epsilon_l > 0$  and  $\epsilon_a > 0$ . Then, the observer error dynamic can be written as

$$\begin{aligned} \dot{\tilde{e}}_1 &= \tilde{e}_2 - \frac{H_1}{\epsilon_l} \tilde{e}_1, \dot{\tilde{e}}_2 = \mathcal{W}_1(\hat{y}_2, \hat{U}_L) - \frac{H_2}{\epsilon_l^2} \tilde{e}_1, \\ \dot{\tilde{e}}_3 &= \tilde{e}_4 - \frac{H_3}{\epsilon_a} \tilde{e}_1, \dot{\tilde{e}}_4 = \mathcal{W}_2(\hat{y}_3, \hat{y}_4, \hat{U}) - \frac{H_4}{\epsilon_a^2} \tilde{e}_3 \end{aligned} \quad (12)$$

with  $\tilde{e}_1 = (y_1 - \hat{y}_1)$ ,  $\tilde{e}_2 = (y_2 - \hat{y}_2)$ ,  $\tilde{e}_3 = (y_3 - \hat{y}_3)$  and  $\tilde{e}_4 = (y_4 - \hat{y}_4)$ . Using (11), we design the following output feedback based nonlinear control for the linear and angular dynamic of the vehicle

$$\begin{aligned} \hat{U}_L &= \beta^{-1}(\ddot{x}_{1d} + k_d \hat{x}_2 + \lambda_L \hat{e}_2 + \delta_l + k_L \hat{S}_L - \hat{u}_l) \\ \hat{u}_l &= \hat{\theta}_1 \text{sign}(\hat{S}_L), \dot{\hat{\theta}}_1 = -\Gamma_1 \text{sign}^T(\hat{S}_L) \hat{S}_L \end{aligned} \quad (13)$$

$$\begin{aligned} \hat{U} &= \mathcal{M}^{-1}(\ddot{x}_{3d} + \lambda_A \hat{e}_4 + k_A \hat{S}_A - \hat{u}_a) \\ \hat{u}_a &= \hat{\theta}_2 \text{sign}(\hat{S}_A), \dot{\hat{\theta}}_2 = -\Gamma_2 \text{sign}^T(\hat{S}_A) \hat{S}_A \end{aligned} \quad (14)$$

Using new variables  $\epsilon_l \eta_1 = \tilde{e}_1$ ,  $\eta_2 = \tilde{e}_2$ ,  $\epsilon_a \eta_3 = \tilde{e}_3$  and  $\eta_4 = \tilde{e}_4$ , we can re-write the observer error model in singularly perturbed form as

$$\begin{aligned} \epsilon_l \dot{\eta}_l &= A_l \eta_l + \epsilon_l B_l \mathcal{W}_l(\epsilon_l, \epsilon_l, \eta_l) \\ \epsilon_a \dot{\eta}_a &= A_a \eta_a + \epsilon_a B_a \mathcal{W}_a(\epsilon_a, \epsilon_a, \eta_a) \end{aligned} \quad (15)$$

where  $\eta_l = [\eta_1, \eta_2]^T$ ,  $\eta_a = [\eta_3, \eta_4]^T$ ,  $\epsilon_l = [e_1, e_2]^T$ ,  $\epsilon_a = [e_3, e_4]^T$ ,  $e_1 = (y_{1d} - y_1)$ ,  $e_2 = (y_{2d} - y_2)$ ,  $e_3 = (y_{3d} - y_3)$ ,  $e_4 = (y_{4d} - y_4)$  with  $y_{1d} = \mathcal{K}_d$ ,  $y_{2d} = \dot{\mathcal{K}}_d$ ,  $y_{3d} = \Theta_d$  and  $y_{4d} = \dot{\Theta}_d$ ,  $A_l = \begin{bmatrix} -H_1 & I_{n \times n} \\ -H_2 & 0_{n \times n} \end{bmatrix}$ ,  $A_a = \begin{bmatrix} -H_3 & I_{n \times n} \\ -H_4 & 0_{n \times n} \end{bmatrix}$ ,  $B_l = B_a = \begin{bmatrix} 0_{n \times n} \\ I_{n \times n} \end{bmatrix}$ . Then, the closed loop system with output feedback control algorithm together with observer can be written as

$$\begin{aligned} \dot{e}_l &= \mathcal{W}_{ls}(e_l, \epsilon_l, \eta_l) \\ \dot{e}_a &= \mathcal{W}_{as}(e_a, \epsilon_a, \eta_a) \end{aligned} \quad (16)$$

where  $\mathcal{W}_{ls}(e_l, \epsilon_l, \eta_l) = [e_2; \mathcal{W}_l(e_l, \epsilon_l, \eta_l)]$  and  $\mathcal{W}_{as}(e_a, \epsilon_a, \eta_a) = [e_4; \mathcal{W}_a(e_a, \epsilon_a, \eta_a)]$ . Then, we can state the following results for output feedback based design.

**Theorem 2:** *There exists positive definite constant diagonal matrices  $k_A$ ,  $k_L$ ,  $k_d$ ,  $\lambda_A$ ,  $\lambda_L$ ,  $\Gamma_1$  and  $\Gamma_2$  such that all the*

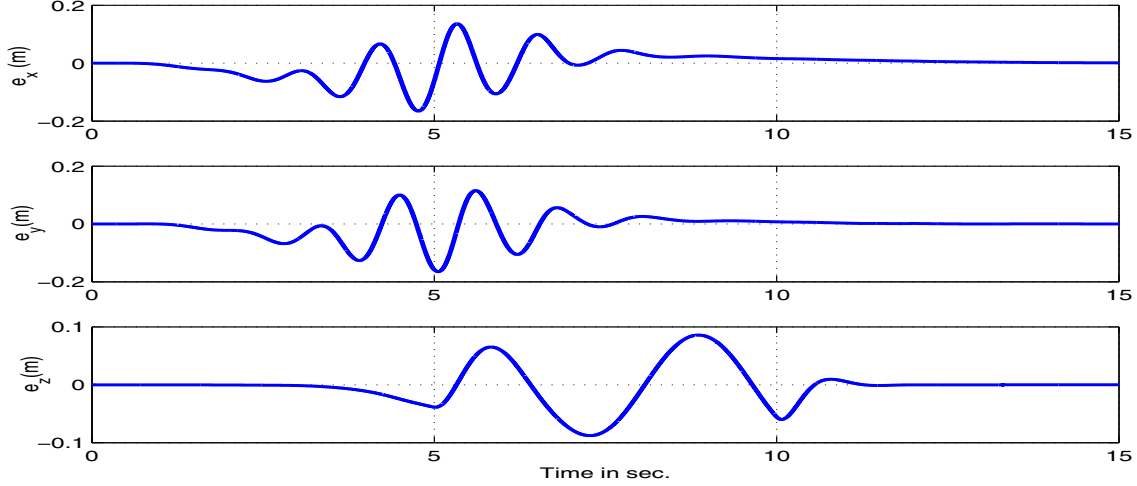


Fig. 1. Position tracking errors in indoor flying environment under nonlinear robust adaptive control algorithm.

error states in closed loop system (16) with the control input (13), (14) are bounded and their bounds converge to the bound achieved under state feedback based design (6)-(9) as  $\epsilon_l \rightarrow 0$  and  $\epsilon_a \rightarrow 0$ .

### III. EVALUATION RESULTS

Let us now implement the proposed design on commercial quadrotor UAV system [1]. We decompose proposed control design into two loops as inner and outer loop. The altitude of the vehicle is measured by barometric pressure sensor. To measure the position of the vehicle in indoor scenario, we use visual tracker provided by Phoenix Technologies Inc. The angular velocity of the vehicles is obtained by using onboard gyroscope. The barometric pressure sensor is used to measure the height of the vehicle. The physical parameters and details about the platform can be found in [1]. The desired trajectories for  $x_d$ ,  $y_d$  and  $\psi_d$  are chosen as the output response of the following transfer function

$$G(s) = \frac{1}{(s+1)^6} \quad (17)$$

The desired trajectory for take-off, cruise flight and landing flights is also generated by above transfer function. The desired roll and pitch trajectories  $\phi_d$  and  $\theta_d$  are designed by using virtual control input vectors  $U_{lx}$  and  $U_{ly}$ . To examine the robustness of the proposed design, the bounds on  $\mathcal{H}_a$  and  $\Pi_t$  are considered as  $\mathcal{H}_a = \Pi_t = 25\sin(t)$  from 5 sec. to 10 sec. Then, the control design parameters are selected as  $k_d = \text{diag}(5, 5, 5)$ ,  $\lambda_A = \text{diag}(20, 20, 20)$ ,  $k_L = \text{diag}(3, 3, 5)$ ,  $\Gamma_1 = \text{diag}(0.005, 0.005, 0.005)$ ,  $k_A = \text{diag}(80, 80, 80)$ ,  $\Gamma_2 = \text{diag}(80, 80, 80)$  and  $\lambda_L = \text{diag}(2.5, 2.5, 5)$ . The observer design parameters are chosen as  $H_i = 10I_{3 \times 3}$  with  $i = 1, 2, 3, 4$ ,  $\epsilon_l = 0.005$  and  $\epsilon_a = 0.005$ . The design parameters are selected by increasing their values from zero to until small variation with tracking performance is observed.

With this above set up, we now implement the proposed

nonlinear output feedback design in the presence of given bounded uncertainty. The results are presented in Figs. 1 to 3. The position tracking errors are given in Fig. 1. Fig. 2 shows the time history of the attitude tracking error. The control inputs profile are shown in Fig. 3. From Figs. 1 to 2, it is noticed that the position and attitude dynamic of the quadrotor UAV oscillates (from 5 sec. to 15 sec.) due to the presence of the input disturbances. The oscillation of the position and attitude dynamics damped out quickly as soon as the disturbance disappeared. The design achieves good tracking performance due to faster response of the inner-loop control design. As a result, the tracking error for both position and attitude dynamics converge quickly closed to zero once the disturbance is vanished. Notice also that the control inputs exhibit small chattering phenomenon with smaller values of steady state tracking error due to the presence of the  $\text{sgn}()$  function and measurement noise associated with the inertial measurement unit. This chattering phenomenon can be removed by estimating  $\text{sgn}()$  function through  $\tanh()$  with the small values of  $\epsilon_{o1}$  and  $\epsilon_{o2}$ .

### IV. CONCLUSION

In this paper, we have addressed output feedback based nonlinear adaptive control problem for MUAV system. First, we have designed state feedback based control provided that all the states of the vehicle dynamic are available for feedback. Then, observer used to estimate the unknown states in linear and angular dynamic to develop output feedback nonlinear tracking control system. Convergence analysis has been given by using Lyapunov method. It has been shown in our analysis that the tracking performance with state feedback design can be recovered asymptotically by output feedback based design. Evaluation results on quadrotor UAV have been used to illustrate the theoretical development of the proposed design.

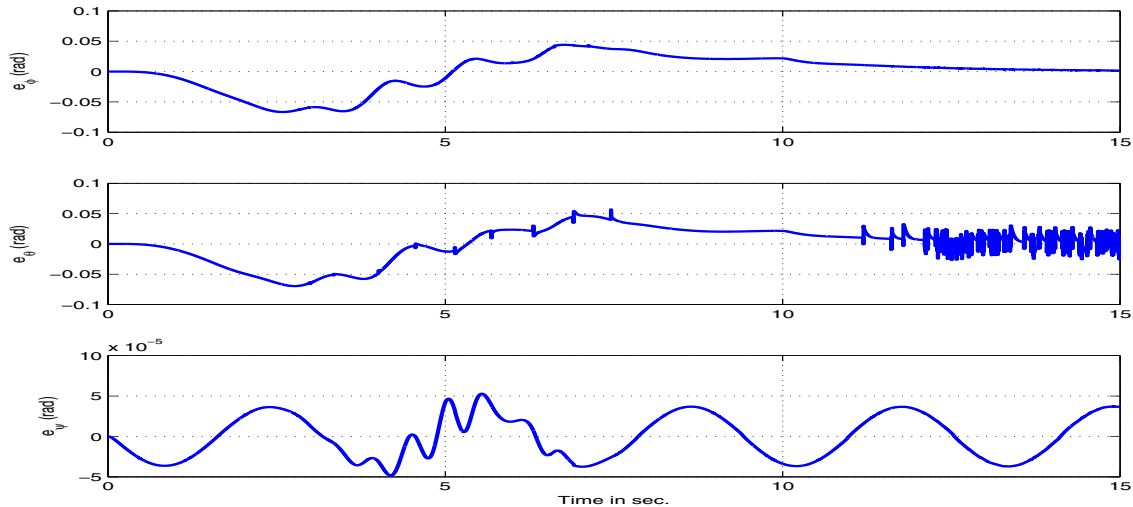


Fig. 2. Attitude tracking errors in indoor flying environment with nonlinear robust adaptive control.

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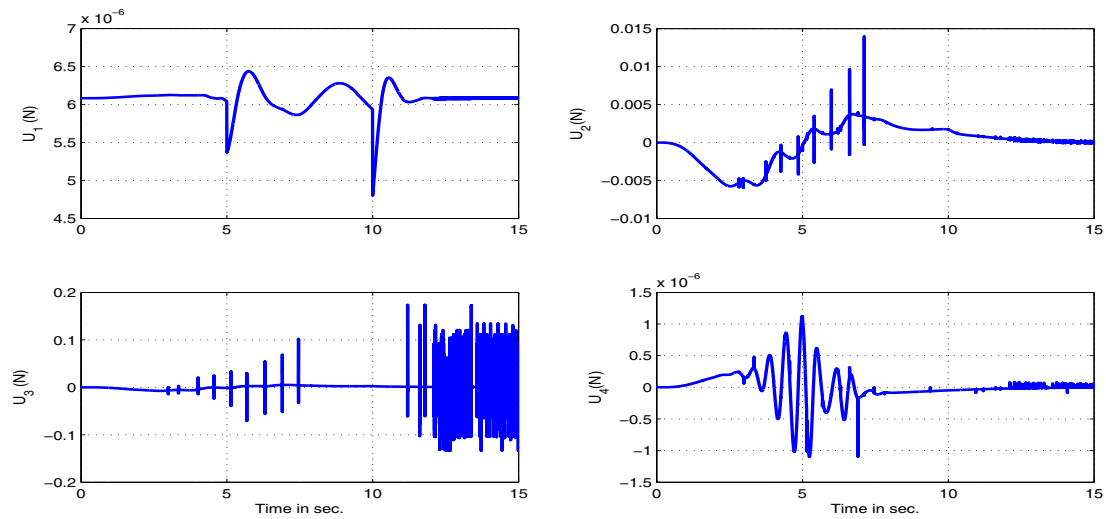


Fig. 3. Control inputs for indoor flying environment with proposed nonlinear robust adaptive control.

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