Stable Indirect Adaptive Predictive Fuzzy Control for Industrial Processes

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Abstract

The paper proposes a stable indirect adaptive fuzzy predictive control, which is based on a discrete-time Takagi-Sugeno (T-S) fuzzy model and on the Generalized predictive control (GPC) algorithm. The T-S fuzzy model is used to approximate the unknown nonlinear plant, that to provide good accuracy in identification of unknown model parameters, three online adaptive laws are proposed. It is demonstrated that the tracking error remains bounded. The stability of closed-loop control system is studied and proved via the Lyapunov stability theory. To validate the theoretical developments and to demonstrate the performance of the proposed control, the controller is applied on a nonlinear simulated laboratory-scale liquid-level process. The simulation results show that the proposed method has a good performance and disturbance rejection capacity in industrial processes.

1 Introduction

Model predictive control (MPC) has been recognized as a powerful methodology for controlling a wide class of nonlinear systems. MPC is a control method that utilizes an explicit process model to predict the future response of a plant by minimizing an objective function. MPC based on linear models is a mature control technique with applications in the industrial processes. And, recently, the control of complex nonlinear industrial processes with nonlinear predictive control has received considerable attention in industrial control community.

One of the most popular and powerful MPC methods applied in industry has been the Generalized predictive control (GPC) [2]. The GPC has been applied in various plants, and has shown good performance results [3], [12]. However, most plants where GPC has been applied were linear.

Previous research has presented GPC for nonlinear plants [10], [17]. The first proposed approach was the linearisation of the model plant [10], [17]. However, this approach may not predict accurately because the operating point may change and the predictor does not remain valid. Furthermore, a disadvantage of GPC, being also a common characteristic to all MPCs, is its assumption of the knowledge of an accurate model.

This assumption may present problems because many complex plants are difficult to model mathematically based in physical laws, or have large uncertainties and strong nonlinearities. An alternative to modeling nonlinear plants are models based on fuzzy logic systems, theoretically supported by the fact that fuzzy logic systems are universal approximators [13], [7]. Takagi-Sugeno (T-S) fuzzy models [11] are suitable for modelling a large class of nonlinear systems and have gained much popularity because of their rule consequent structure which is a mathematical function. In off-line training algorithms the discrete-time T-S fuzzy model can be obtained from input-output data collected from a plant. Moreover, the time-variant behavior of the controlled plant, usually caused by disturbances or varying operating points and parameters of the model, must be considered in the process model. This motivates the introduction of adaptive control methodologies to solve the problem.

This paper focuses on indirect adaptive control, where the model parameters of T-S fuzzy model are initially constructed from human knowledge about the unknown plant, and then iteratively adjusted on-line to reduce the output error between the plant and a reference model. The method can operate either with or without integrating initial human knowledge about the plant.

In [14] a general formulation is introduced for an actuator arrearage faults. Based on this formulation, an improved generalized predictive control scheme is proposed for a class of single-input single-output linear systems. However, there are many applications in industry where plant models are nonlinear and it does not consider on-line auto-adaptation mechanisms to take into account and overcome complex and/or unknown time-varying plant behavior and system disturbances, and in particular to improve performance under such conditions.

In [5], together with the neurofuzzy modelling using T-S fuzzy model, several strategies based on nonlinear predictive control are presented. The low computational cost associated with neurofuzzy models and controllers makes them suitable candidates to be implemented into
industrial Programmable Logic Controllers. In [8] hybrid MPC methods are developed to solve a class of practical nonlinear control problems by approximating the nonlinear system with a piecewise affine hybrid model; And an experimental case-study on a vacuum chamber of a wire-annealing machine is presented. In [15] is investigated the problem of constrained MPC for T-S fuzzy system with parallel-distributed compensation (PDC) and non-PDC law. New sufficient conditions are proposed in terms of linear-matrix inequalities (LMIs). However, these methods require some knowledge about the model of the process to be controlled, in the form of a model close to the real model of the process. Such knowledge can be difficult to extract in complex industrial processes.

In [6] an application of fuzzy predictive control to a solar power plant is presented. The proposed predictive controller uses fuzzy characterization of goals and constraints, based on the fuzzy optimization framework for multi-objective satisfaction problems. However, only heuristic control has been done, stability has not been an issue.

In this paper, a new stable indirect adaptive fuzzy model-based predictive controller is proposed for a class of nonlinear discrete-time processes. The proposed controller is based on a GPC algorithm and uses a discrete-time T-S fuzzy model adapted on-line, and is able to ensure that the tracking error remains bounded. The stability of the closed-loop control system is studied and proved via the Lyapunov stability theory. A diagram of the proposed adaptive fuzzy generalized predictive control (AFGPC) approach is represented in Fig. 1. As can be seen, the control scheme consists of the plant, the controller, and the adaptive T-S fuzzy model. The proposed controller, is composed by a model-based predictive control whose model parameters are adjusted on-line by adaptation laws based on Lyapunov Theory. In the presently proposed method, human knowledge or a physical model about the plant model are not necessary. Nevertheless, initial human knowledge about the plant can be integrated in the method.

The paper is organized as follows. Section 2 presents a nonlinear systems modeling method using T-S fuzzy models. Section 3 presents a brief overview of GPC. The proposed control method is described in Section 4. In Section 5, the results of simulations are presented and analysed. Finally, Section 6 makes concluding remarks.

2 Nonlinear Systems Modeling Using T-S Fuzzy Models

Takagi-Sugeno (T-S) fuzzy models are universal approximators capable of approximating any continuous nonlinear system [16]. A large class of nonlinear processes can be represented by the following NARX model:

\[ y(k+1) = f(\xi(k), u(k)). \] (1)

where \( u(\cdot): \mathbb{N} \rightarrow \mathbb{R} \) and \( y(\cdot): \mathbb{N} \rightarrow \mathbb{R} \) are the process input and output, and the regression vector \( \xi(k) \) includes current and lagged outputs and inputs:

\[ \xi(k) = [y(k), \ldots, y(k-n_y), u(k-1), \ldots, u(k-n_u)] \] (2)

where \( n_u \in \mathbb{N} \) and \( n_y \in \mathbb{N} \) are the orders of input and output, respectively. In the discrete-time nonlinear SISO plant, \( f(\cdot): \mathbb{R}^{n_u+n_y+1} \rightarrow \mathbb{R} \) represents a nonlinear mapping which is assumed to be unknown. \( f(\cdot) \) will be approximated by a T-S fuzzy system. To design the T-S fuzzy model, the global operation of the nonlinear system (1) can be accurately approximated by a set of local affine models. Thus, system (1) can be described by a T-S fuzzy model defined by the following fuzzy rules:

\[ \begin{align*}
R_i: & \quad \text{IF } x_1(k) \text{ is } A^i_1 \text{ and } \ldots \text{ and } x_n(k) \text{ is } A^i_n \text{ THEN } y_i(k+1) = a_i(z^{-1})y(k) + b_i(z^{-1})u(k), \\
& \quad i = 1, \ldots, N, \end{align*} \] (3)

where \( R_i \) \((i = 1, 2, \ldots, N)\) represents the \( i \)-th fuzzy rule, \( N \) is the number of rules,

\[ a_i(z^{-1}) = a_{i1} + a_{i2}z^{-1} + \ldots + a_{in}z^{-n_i}, \]

\[ b_i(z^{-1}) = b_{i1} + b_{i2}z^{-1} + \ldots + b_{in}z^{-n_i-1}, \] (4)

and \( u(k) \) is the control output. \( x_1(k), \ldots, x_n(k) \) are the input variables of the T-S fuzzy system - they can be any variables chosen by the designer [e.g. \( y(k-1), u(k-1), \) or other]. \( A^i_j \) are linguistic terms characterized by fuzzy membership functions \( \mu_{A^i_j}(x_i) \) which describe the local operating regions of the plant. Thus, from (3) \( y(k+1) \) can be rewritten as

\[ \begin{align*}
y(k+1) &= \sum_{i=1}^{N} \omega^i(x(k))\cdot [a_i(z^{-1})y(k) + b_i(z^{-1})u(k)], \\
&= \sum_{i=1}^{N} \omega^i(x(k))\cdot \theta^i_1Y(k) + \sum_{i=1}^{N} \omega^i(x(k))\cdot \theta^i_2X_u(k) \\
&\quad + \sum_{i=1}^{N} \omega^i(x(k))\cdot \theta^i_3u(k), \\
&= \Theta_f\cdot \Psi_f(k) + \Theta_{f2}\cdot \Psi_{f2}(k) + \Theta_{f3}\cdot \Psi_{f3}(k), \end{align*} \]

\[ = f_1(x(k), Y(k))\cdot \Theta_{f1} + f_2(x(k), X_u(k))\cdot \Theta_{f2} + g(x(k))\cdot \Theta_{f3}, \] (5)
Assumption 1 There exist (optimal) model parameters vectors $\Theta^*_f$, $\Theta^*_g$ and $\Theta^*_y$ that makes T-S fuzzy model (5) become a perfect representation of the real plant (1).

Taking into account Assumption 1, i.e. assuming there is no modeling error, and using (5), then the real plant (1) can be represented as

$$y(k + 1) = \Theta_f^* \Psi_f(k) + \Theta_g^* \Psi_g(k)u(k). \quad (12)$$

It is assumed that the parameters vectors $\Theta_f^*$, $\Theta_g^*$ and $\Theta_y^*$ in (12) are unknown. Thus, an approximate model for $y(k + 1)$ is defined as

$$\hat{y}(k + 1) = \Theta_f^* \Psi_f(k) + \Theta_g^* \Psi_g(k)u(k). \quad (13)$$

Assumption 2 The optimal fuzzy approximator errors, satisfies the followings inequalities, where $\beta_{f1}$, $\beta_{f2}$ and $\beta_g$ are small positives values,

$$\sup_{\hat{x} \in \mathbb{X}} \left\| \Theta_f^* \Psi_f(k) - (\Theta_f^*)^T \Psi_f(k) \right\| \leq \beta_{f1}, \quad (14)$$

$$\sup_{\hat{x} \in \mathbb{X}} \left\| \Theta_g^* \Psi_g(k) - (\Theta_g^*)^T \Psi_g(k) \right\| \leq \beta_g. \quad (16)$$

3 Predictive Control Law

The adaptive fuzzy generalized predictive control (AFGPC) developed in this paper is motivated from the GPC strategy [2]. For completeness this section briefly overviews the GPC. It is assumed that the plant model is of the form (5), which can be rewritten as follows:

$$\hat{a}(z^{-1})y(k+1) = \hat{b}(z^{-1})u(k), \quad (17)$$

where

$$\hat{a}(z^{-1}) = 1 - \hat{a}_1 z^{-1} - \ldots - \hat{a}_n z^{-n}, \quad (18)$$

$$\hat{b}(z^{-1}) = \hat{b}_1 + \hat{b}_2 z^{-1} + \ldots + \hat{b}_n z^{-(n-1)}, \quad (19)$$

$$\hat{a}_j = \sum_{i=1}^{N} \omega^j(x(k))a_{ji}, \quad \hat{b}_j = \sum_{i=1}^{N} \omega^j(x(k))b_{ji}. \quad (20)$$

The GPC control laws is obtained to minimize the following cost function

$$J(k) = \sum_{p=1}^{N_p} \left[ \hat{y}(k + p + 1|k) - \phi_p r(k + p + 1) \right]^2 + \sum_{p=1}^{N_u} |\lambda(p)\Delta u(k + p - 1)|^2, \quad (21)$$

where $\hat{y}(k + p + 1|k)$ is an optimal p-step ahead prediction of the system on instant $k$, $r(k + p + 1)$ is the future reference trajectory, $\phi_p$ is the feed forward gain for $p$ steps ahead, $\Delta = 1 - z^{-1}$, and $\lambda(p)$ is a weighting sequence that considers the future behaviour. $N_p$ and $N_u$ are output and control horizons, respectively. Consider the following Diophantine equation (22):

$$1 = \Delta e_p(z^{-1})\hat{a}(z^{-1}) + z^{-p}f_p(z^{-1}), \quad (22)$$

$$e_p(z^{-1}) = 1 + e_{p,1}z^{-1} + \ldots + e_{p,z^{-p}}, \quad (23)$$

$$f_p(z^{-1}) = f_{p,0} + f_{p,1}z^{-1} + \ldots + f_{p,n_v}z^{-n_v}, \quad (24)$$

where $e_p(z^{-1})$ and $f_p(z^{-1})$ can be obtained dividing 1 by $\Delta \hat{a}(z^{-1})$ until the remainder can be factorized as $z^{-p}f_p(z^{-1})$ (see [2] for more details). The quotient of the division is the polynomial $e_p(z^{-1})$. Multiplying (17) by $\Delta z^p e_p(z^{-1})$ yields

$$\Delta z^p e_p(z^{-1})\hat{a}(z^{-1})\hat{y}(k + 1) = \Delta z^p e_p(z^{-1})\hat{b}(z^{-1})u(k). \quad (25)$$

Defining

$$g_p(z^{-1}) = e_p(z^{-1})\hat{b}(z^{-1}), \quad (26)$$

and substituting (22) and (26) in (25) yields

$$\hat{y}(k + p + 1|k) = f_p(z^{-1})u(k + 1) + g_p(z^{-1})\Delta u(k + p - 1). \quad (27)$$

Thus, the best prediction of $y(k + p + 1|k)$ is

$$\hat{y}(k + p + 1|k) = f_p(z^{-1})u(k + 1) + g_p(z^{-1})\Delta u(k + p - 1). \quad (28)$$
A simple and efficient way to obtain the polynomials $e_p(z^{-1})$ and $f_p(z^{-1})$ is to use recursion of the Diophantine equation that has been demonstrated in [4]. If we want to reduce computation costs, $N_u = 1$ is chosen, thus $\lambda(p)$ in (21) is simplified to $\lambda$, and is considered $\Delta u(k+1) = \ldots = \Delta u(k+N_u-1) = 0$. Thus, rewritten (28) as

$$y(k+2) = G_1u(k) + F(z^{-1})y(k+1) + L(z^{-1}),$$  

where

$$y(k+2) = \begin{bmatrix} \hat{y}(k+2) \\ \hat{y}(k+3) \\ \vdots \\ \hat{y}(k+N_p+1) \end{bmatrix}, L = \begin{bmatrix} g_1(z^{-1}) - \hat{g}_1(z^{-1}) & z\Delta u(k-1) \\ g_2(z^{-1}) - \hat{g}_2(z^{-1}) & z\Delta u(k-1) \\ \vdots \\ g_{N_p}(z^{-1}) - \hat{g}_{N_p}(z^{-1}) & z^{N_p}\Delta u(k-1) \end{bmatrix},$$

$$\hat{g}_p(z^{-1}) = g_{p,0} + g_{p,1}z^{-1} + \ldots + g_{p,p-1}z^{-(p-1)} \quad (31)$$

$$G_1 = \begin{bmatrix} g_{1,0} & g_{1,1} & \ldots & g_{1,N_p,N_p-1} \end{bmatrix}^T.$$  

Using (29), (21) can be rewritten as

$$J_{eq}(k) = [Fy(k+1) + G_1\Delta u(k) + L - \Phi R]^T [Fy(k+1) + G_1\Delta u(k) + L - \Phi R] + [\lambda \Delta u(k)]^2,$$  

where

$$\Phi = \text{diag} \{ \phi_{d+1}, \phi_{d+2}, \ldots, \phi_{N_p} \},$$

$$R = [r(k+d+1), \ldots, r(k+N_p)]^T.$$  

By minimizing $J_{eq}(k)$, the following optimum control increment is obtained (see [2]):

$$\Delta u(k) = K[\Phi R - Fy(k+1) - L]$$  

where $K = (G_1^T G_1 + \lambda^{-1})^{-1}$, and $u(k) \in [u_{Min}, u_{Max}]$.

### 4 Adaptive Predictive Fuzzy Control

This section explains how the T-S fuzzy model (Sections 2) will be used in the predictive control law (Section 3) such that the model parameters can be adapted in novel Adaptive Predictive Fuzzy Control framework. Taking into account a nonlinear discrete-time dynamic system model and the predictive control law, the closed-loop dynamic error equation will be determined. The next step will be to choose adaptive laws with the goal of minimizing the tracking error $\epsilon$ and the parameters errors $\Theta_{f1}$, $\Theta_{f2}$ and $\Theta_g$ by the minimization of a candidate Lyapunov function.

Assumption 3 [9] $|g(x(k)|\Theta_g)| > \delta$, where $\delta$ is a small real positive number, which implies that the relative degree of the T-S fuzzy model and, consequently, the relative degree of the plant are both equal to one.

Without loss of generality, it is assumed that $g(x(k)|\Theta_g) > 0$. To simplify the process of computer calculation it is considered that $g(x(k)|\Theta_g) = g > 0$ is constant.

Assumption 4 [9] The reference trajectory $r(k+1)$ satisfies

$$\|r(k+1)\| \leq U,$$  

where $U$ is a known bound.

For the moment, assume that functions $f_1(x(k), y(k)|\Theta_{f1}), f_2(x(k), x_u(k)|\Theta_{f2})$ and $g(x(k)|\Theta_g) = g$ are known. Let $k = [k_0, \ldots, k_1]^T$ be chosen such that the zeros of polynomial $z^{k} = z^{n} + k_1z^{n-1} + \ldots + k_n$ are inside in the unit circle centered at the origin of the $z$ plane, and choose the control law

$$u_*(k) = \frac{1}{g} [-f_1(x(k), y(k)|\Theta_{f1}) - f_2(x(k), x_u(k)|\Theta_{f2}) + r(k+1) + k^Te(k)],$$  

where $r(k+1)$ is the reference model output signal, and

$$e(k) = (e(k-n-1), \ldots, e(k-1), e(k))^T,$$  

$$e(k) = r(k) - y(k).$$

Substituting (40) into (5) and after some manipulation with (38), the following closed-loop dynamic equation is obtained:

$$e(k+1) = -k^Te(k) + g[u_*(k) - u(k)].$$  

Let

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix}, b_g = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix},$$

then (41) can be rewritten into vector form as

$$e(k+1) = \Lambda e(k) + b_g(u_*(k) - u(k)).$$  

Assuming that $f_1(x(k), y(k)|\Theta_{f1}), f_2(x(k), x_u(k)|\Theta_{f2})$ and $g(x(k)|\Theta_g)$ are unknown, using (36) and substituting $y(k+1)$ by its T-S fuzzy approximation $\hat{y}(k+1)$ (13), then the control law is designed as

$$u(k) = u_*(x(k), \Theta_g)$$  

where

$$= K[\Phi R - Fy(k+1) + L] + u(k-1),$$

$$= K[\Phi R - F(\Theta_{f1}(k))\Psi_{f1}(k) + \Theta_{f2}(k)\Psi_{f2}(k) + \Theta_g(k)\Psi_g(k)u(k)) - L] + u(k-1).$$
With a minimum approximation error

\[ u_e(k) \approx u^*(k) = u [x_e(k), \Theta_{f1}, \Theta_{f2}, \Theta_g] \]  

(45)

where \( u^*(k) \) is defined as the optimal command. By substituting \( u^*(k) \) for \( u_e(k) \), (43) is rewritten as

\[
\begin{aligned}
\varepsilon(k+1) &= \Lambda \varepsilon(k) + b_g [u^*(k) - u(k)] , \\
\varepsilon(k) &= \Lambda \varepsilon(k) + b_g [\Theta_{f1}(k) \Psi_{f1}(k) + \Theta_{f2}(k) \Psi_{f2}(k) + \Theta_g(k) \Psi_g(k) u(k)] 
\end{aligned}
\]

(46)

where \( \hat{\Theta}_{f1}(k) = \Theta_{f1}(k) - \Theta_{f1}^{g} \), \( \hat{\Theta}_{f2}(k) = \Theta_{f2}(k) - \Theta_{f2}^{g} \) and \( \hat{\Theta}_g(k) = \Theta_g(k) - \Theta_g^{g} \).

**Assumption 5 [9]** There exist \( \alpha > 0 \) and a positive-definite symmetric matrix \( \mathbf{P} \) such that for matrix \( \Lambda (42), \)

\[
\Lambda^T \mathbf{P} \Lambda - \mathbf{P} \leq -\alpha \mathbf{I} < 0.
\]

(47)

Consider the candidate Lyapunov function for system (46),

\[
V(k) = \frac{1}{2} \epsilon^T(k) \epsilon(k) + \frac{1}{2 \gamma_{f1}} \hat{\Theta}_{f1}^T(k-1) \hat{\Theta}_{f1}(k-1) + \frac{1}{2 \gamma_{f2}} \hat{\Theta}_{f2}^T(k-1) \hat{\Theta}_{f2}(k-1) + \frac{1}{2 \gamma_g} \hat{\Theta}_g^T(k-1) \hat{\Theta}_g(k-1),
\]

(48)

where \( \gamma_{f1}, \gamma_{f2} \) and \( \gamma_g \) are positive constants and \( \mathbf{P} \) is a positive-definite symmetric \( n \times n \) matrix.

In order to minimize the tracking error \( \epsilon(k) \) and the parameter errors \( \hat{\Theta}_{f1}, \hat{\Theta}_{f2} \) and \( \hat{\Theta}_g \), equation (48) will be minimized. To decrease \( V(k) \) it is necessary ensure that \( \Delta V(k) < 0 \). \( \Delta V(k) \) will be analysed and calculated in (57)-(63).

Taking the first time difference of (48) and with some manipulations (58) is deduced. Using (46) and defining \( \rho = b_g \mathbf{K} \Phi_{f1}(k) \Psi_{f1}(k) + \Theta_{f2}(k) \Psi_{f2}(k) + \Theta_g(k) \Psi_g(k) u(k) \), (59) can be obtained. By Assumption 5, (60) can be written. Since \( \mathbf{P} \) is symmetric, then \( \rho^T \mathbf{P} \epsilon(k) = [\epsilon(k)]^T \mathbf{P} \rho \) and with some manipulations (62) is derived. Then, with some manipulations (63) is obtained. To minimize \( V(k) \), following parameters adaptation laws are chosen such that third, fourth and fifth terms in (63) are zero. Thus are obtained the following adaptation laws:

\[
\begin{aligned}
\hat{\Theta}_{f1}(k) &= \hat{\Theta}_{f1}(k-1) - \gamma_{f1} [\epsilon(k)]^T \mathbf{P} b_g \Phi_{f1}(k), \\
\hat{\Theta}_{f2}(k) &= \hat{\Theta}_{f2}(k-1) - \gamma_{f2} [\epsilon(k)]^T \mathbf{P} b_g \Phi_{f2}(k), \\
\hat{\Theta}_g(k) &= \hat{\Theta}_g(k-1) - \gamma_g [\epsilon(k)]^T \mathbf{P} b_g \Phi_{g}(k) u(k),
\end{aligned}
\]

(49), (50) and (51), equation (63) can be rewritten as

\[
\Delta V(k) \leq \frac{1}{2} \alpha \epsilon^T(k) \epsilon(k) + \frac{1}{2} \rho^T \mathbf{P} \rho \leq \alpha \| \epsilon(k) \|^2 + \sigma_{max}(\mathbf{P}) \| \rho \|^2,
\]

(52)

where \( \sigma_{max}(\mathbf{P}) \) is the largest singular value of \( \mathbf{P}, \alpha = 0/2, \) and

\[
\| \rho \|^2 = \| b_g \mathbf{K} \Phi_{f1}(k) \Psi_{f1}(k) + \hat{\Theta}_{f2}(k) \Psi_{f2}(k) + \hat{\Theta}_g(k) \Psi_g(k) u(k) \| \leq \| b_g \| \| \mathbf{K} \| \| \Phi_{f1}(k) \| \| \Psi_{f1}(k) \| \| \hat{\Theta}_{f2}(k) \| \| \Psi_{f2}(k) \| \| \hat{\Theta}_g(k) \| \| \Psi_g(k) \| \| u(k) \|. 
\]

(53)

Using Assumption 1, (53) is rewritten as

\[
\| \rho \| \leq \| b_g \| \| \mathbf{K} \| \| \beta_{f1} + \| b_g \| \| \mathbf{K} \| \| \beta_{f2} + \| b_g \| \| \mathbf{K} \| \| \beta_g \| \| u(k) \|. 
\]

(54)

From Assumption 2, the model parameters, i.e. the components of \( \hat{\Theta}_{f1}, \hat{\Theta}_{f2} \) and \( \hat{\Theta}_g \), are bounded. Thus, from (7)-(11), (18)-(20), (24), (26), and (32), all the elements of \( \mathbf{F} \) and \( \mathbf{G} \) are bounded.

Consequently, from (42), (54), there exists a positive constant \( \rho_c \) such that

\[
\| \rho \| \leq \rho_c.
\]

(55)

From (52) and (55), it is concluded that \( \Delta V(k) \leq 0 \) outside the ball

\[
\left\{ \epsilon(k) : \| \epsilon(k) \| < \epsilon \sqrt{\frac{\sigma_{max}(\mathbf{P})}{\alpha} \rho_c} \right\}.
\]

(56)

**Theorem 1** Consider the closed loop system consisting of the plant (12), controller (36) and parameter adaptation laws (49), (50) and (51). If Assumptions 1-5 hold, then the plant tracking error vector \( \epsilon(k) \) is bounded above by \( \epsilon \) defined in (56).

The proof of Theorem 1 is given by the above analysis and by (56).

Algorithm 1 summarizes the design and operation of the proposed adaptive fuzzy generalized predictive control method.

It should be noted that although this control algorithm has five assumptions, Assumptions 1, 3, 4, and 5 are very feasible to be implemented. Moreover, the proposed adaptation laws ensure that Assumption 2 is met.

**5 Simulation Results**

This section presents simulation results to validate the theoretical developments and to demonstrate the performance of the proposed adaptive predictive fuzzy control scheme in nonlinear systems. The control of a laboratory-scale liquid-level process will be simulated. In this simulation, to test the reference tracking performance, parameters convergence, and disturbance rejection capacity, the reference input \( r(k) \) is changed with time and a load disturbance \( u(k) \) is applied.
\[
\Delta V(k) = V(k + 1) - V(k), \\
= \frac{1}{2} e^T(k + 1) Pe(k + 1) - \frac{1}{2} e^T(k) Pe(k) + \frac{1}{2} \gamma_{f1}^T \hat{\Theta}_{f1}(k) \hat{\Theta}_{f1}(k) + \frac{1}{2} \gamma_{f2}^T \hat{\Theta}_{f2}(k) \hat{\Theta}_{f2}(k) + \frac{1}{2} \gamma_{y1}^T \hat{\Theta}_{y1}(k) \hat{\Theta}_{y1}(k) + \frac{1}{2} \gamma_{y2}^T \hat{\Theta}_{y2}(k) \hat{\Theta}_{y2}(k), \\
= \frac{1}{2} e^T(k + 1) Pe(k + 1) - \frac{1}{2} e^T(k) Pe(k) + \frac{1}{2} \gamma_{f1}^T \left[ \Theta_{f1}(k) - \Theta_{f1}(k - 1) \right]^T \left[ \Theta_{f1}(k) - \Theta_{f1}(k - 1) \right] \\
+ \frac{1}{2} \gamma_{f2}^T \left[ \Theta_{f2}(k) - \Theta_{f2}(k - 1) \right]^T \left[ \Theta_{f2}(k) - \Theta_{f2}(k - 1) \right] \\
+ \frac{1}{2} \gamma_{y1}^T \left[ \Theta_{y1}(k) - \Theta_{y1}(k - 1) \right]^T \left[ \Theta_{y1}(k) - \Theta_{y1}(k - 1) \right] \\
+ \frac{1}{2} \gamma_{y2}^T \left[ \Theta_{y2}(k) - \Theta_{y2}(k - 1) \right]^T \left[ \Theta_{y2}(k) - \Theta_{y2}(k - 1) \right] \\
+ \frac{1}{2} \gamma_{y2}^T \left[ \Theta_{y2}(k) - \Theta_{y2}(k - 1) \right]^T \left[ \Theta_{y2}(k) - \Theta_{y2}(k - 1) \right] \\
= \frac{1}{2} e^T(k) \left( \Lambda^TPA - P \right) e(k) + \rho^T PAe(k) - \frac{1}{2} \gamma_{f1}^T \left[ \Theta_{f1}(k) - \Theta_{f1}(k - 1) \right]^T \left[ \Theta_{f1}(k) - \Theta_{f1}(k - 1) \right] \\
+ \frac{1}{2} \gamma_{f2}^T \left[ \Theta_{f2}(k) - \Theta_{f2}(k - 1) \right]^T \left[ \Theta_{f2}(k) - \Theta_{f2}(k - 1) \right] \\
+ \frac{1}{2} \gamma_{y1}^T \left[ \Theta_{y1}(k) - \Theta_{y1}(k - 1) \right]^T \left[ \Theta_{y1}(k) - \Theta_{y1}(k - 1) \right] \\
+ \frac{1}{2} \gamma_{y2}^T \left[ \Theta_{y2}(k) - \Theta_{y2}(k - 1) \right]^T \left[ \Theta_{y2}(k) - \Theta_{y2}(k - 1) \right]. \\
\]

In this simulation, the following nonlinear model of a laboratory-scale liquid-level process is considered [1]:

\[
y(k) = 0.97229y(k - 1) + 0.35785u(k - 1) \\
-0.1295u(k - 2) - 0.04228g^2(k - 2) \\
-0.3103u(k - 1)u(k - 1) \\
+0.10639y(k - 2)u(k - 2) \\
-0.02256g^2(k - 1)g^2(k - 2) \\
+0.3084g^2(k - 1)g^2(k - 2) \\
+0.1087g^2(k - 2)u(k - 1)u(k - 2) \\
-0.3513g^2(k - 1)u(k - 2) + v(k), \\
\]

where \(v(k)\) is an external disturbance. The input order and output order are \(n_u = 2\) and \(n_y = 2\), respectively. The following controller parameters were chosen by the user: \(N_p = 15\), \(\lambda = 50\), \(g = 1\), \(k_1 = 0.9\), \(P = 1\), \(\gamma_{f1} = 10\), \(\gamma_{f2} = 1.5\) and \(\gamma_{y} = 1.5\). The reference input was

\[
r(k) = \begin{cases} 
1, & 0 < k \leq 500, \\
0.2, & 500 < k \leq 900, \\
1, & 900 < k \leq 1400, 
\end{cases} 
\]

and the load disturbance was \(v(k) = 0.08\) for \(k \geq 1100\), and \(v(k) = 0\), otherwise. The input variables (6) of the
Algorithm 1 Proposed adaptive fuzzy generalized predictive control algorithm.

1. Design control parameters: \( n_x, n_y, N_p, \lambda, g, k, P, \gamma_1, \gamma_2 \) and \( \gamma_3 \).
2. Construct the fuzzy rule base: input variables, respective membership functions and the fuzzy rules.
3. (Initialization) Initialize the model parameters (9), (10) and (11) (with or without human knowledge about the plant model) and initialize \( u(0) \).
4. Compute \( \bar{a}(z^{-1}) \) and \( \bar{b}(z^{-1}) \) using (18) and (19), respectively.
5. Compute control signal \( u(k) \) with (36):
   \[
   \Delta u(k) = K(\Phi R - \Phi f(y(k)) - L).
   \]
6. Adapt the T-S fuzzy model parameters \( a_j, b_j \) of (4)) with (49), (50) and (51).
7. Go to step 4.

fuzzy rules were chosen as \( x(k) = [y(k - 1), y(k - 7), u(k - 1)]^T \), where to save the computational cost in each input variable there are 3 membership functions that were designed taking into account the corresponding range, \( y(k - 1), y(k - 7), u(k - 1) \in [-3; 3] \), and was chosen \( \exp(-\frac{(x_1 + 3)^2}{201.5}) \), \( \exp(-\frac{x_2^2}{201.5}) \) and \( \exp(-\frac{(x_3 - 3)^2}{201.5}) \).

The fuzzy rule-base contains rules covering all combinations of membership functions of the 3 input variables, giving a total of \( 3^3 = 27 \) rules. All controller adjustable parameters [consequent parameters of the rules (3), components of (9), (10) and (11),] are initialized to 0.01, to represent the initial absence of knowledge about the plant (64).

From the results shown in Fig. 2 it can be seen that the proposed controller is able to adequately (attain and control the system output at the desired reference \( r(k) \)). In terms of initial response of the controller, it can be observed that although there is no initial model knowledge (parameters initialized 0.01), the controller quickly reaches the desired reference signal. When the load disturbance \( v(k) \) is applied at \( k = 1100 \), there is an overshoot in system response. As can be seen the controller eliminates this disturbance.

The temporal evolution of the adjustable parameters of the controller are shown in Fig. 3. The model parameters, after being initialized with values near zero (0.01), are then adjusted taking in account the desired response. When the load disturbance is applied, the parameters are again adjusted taking into account the corresponding changes in the system. This illustrates that the adaptation mechanisms worked adequately.

6 Conclusion

This paper has proposed a new stable indirect adaptive model-based predictive controller for a class of nonlinear discrete-time process. The proposed controller is based on the GPC algorithm and uses an adaptive T-S fuzzy model that is adapted online. To provide a good accuracy in identification of unknown model parameters, online adaptive laws were proposed. It was demonstrated that the tracking error remains bounded. The stability of closed-loop control system was studied and proved via the Lyapunov stability theory. The simulation results have shown that the proposed method is able to adequately control the plant without human knowledge about the plant model, and has good tracking performance and disturbance rejection capability. This evidence suggests that the proposed controller could be a good option for industrial process control. As can be seen in the simulations, the adjustable parameters are adjusted for control of the unknown plant and taking into account changes in the system.

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References

Figure 2: Results of the control of the laboratory-scale liquid-level process (64): (a) reference signal \( r(k) \), load disturbance \( v(k) \), and output \( y(k) \), and (b) applied command signal \( u(k) \).

Figure 3: Temporal evolution of the adjustable parameters \( a_{1i} \) and \( a_{2i} \); and \( b_{1i} \) and \( b_{2i} \).


