Abstract—A proof of concept of a purely virtual test platform for critical cyberphysical systems in closed-loop is presented in this work. An $H_\infty$ direct adaptive fuzzy controller is formulated to tackle the wheel slip tracking problem in Antilock-Braking System over a CAN network. A Lyapunov function for the nonlinear control system is derived using the Riccati equation solution in order to prove stability and robustness with respect to network-induced delays, data packet losses, and model uncertainty. Simulation results show that high performance and robustness are achieved.

Index Terms—V&V Platform, Antilock-Braking System, Networked Control System, CAN network, $H_\infty$ Control, Direct Adaptive Fuzzy Control

I. INTRODUCTION

During the early stages of development of cyberphysical system (CPS) [1], [2], testing the whole system in its physical environment may not be feasible or even possible due to safety, availability, and costs. Additionally, validating behaviors in error situations often involves performing destructive tests that are either not feasible in a real environment, or are once again too expensive. These limitations can be overcome by the use of simulators that can interact with the real plant and test the behavior of the system [3]. This work presents a proof of concept of KhronoSim1, a distributed platform for testing critical CPSs in closed-loop capable of tackling the above-mentioned issues.

The chosen case study for this proof of concept is an Antilock-Braking System (ABS), which is a critical CPS widely used in both automobiles and aircrafts. Its goal is to improve the braking system by controlling the wheel slip in order to ensure safety by preventing wheel lock up and reducing the stopping distance. The ABS is usually controlled by networked control systems (NCS), where the overall system dynamics emerges from the interaction among physical dynamics, computational dynamics, and communication networks. Such NCS structure paradigm introduces advantages such as high reliability, low cost, power requirements reduction, simplicity in installation and maintenance, decreased wiring, and increased system flexibility, as well as easy resource sharing. However, the insertion of a communication network into a control loop may lead to performance degradation. The main causes are network-induced delays and data packet losses, which occur in a random and time varying fashion during exchanging information among control system components connected to the shared network. To compose the networked system in this case study, the chosen strategy to shared media access was the CAN network because it is widely implemented in automotive and other industrial applications [4].

Some of the above-mentioned difficulties in ABS control design have encouraged the adaptation and implementation of many advanced control techniques [5], [6]. Among these methods, sliding-mode control is commonly used to reduce the dependency on a model [7], [8], [9], [10]. Moreover, a class of fuzzy/neural network controls and their combination with adaptive approaches [11], [12], [13] has been used for adaptive control of wheel slip. In addition, feedback control [14], [15], extremum-seeking control [16], and robust control [17] were also applied in this research field. However, wheel slip control viewed as a NCS has rarely been addressed [18].

In order to effectively control the wheel slip, taking into consideration the network issues, it is proposed in this work a model-free design approach based on stable direct adaptive fuzzy logic control (AFC), where a filtered error is incorporated to facilitate the compensation of time-varying network-induced delay, and an $H_\infty$ auxiliary control is added to compensate the external disturbances and the approximation error. By this proposed $H_\infty$ direct adaptive fuzzy controller ($H_\infty$AFC), the stability of the closed loop system is guaranteed using Lyapunov stability theory, i.e., the system output converges to the reference signal and all the signals of the closed loop systems are bounded.

This paper is organized as follows. Section II provides a brief explanation about the KhronoSim test platform and how it is used in this work. Section III presents the network environment and depicts its impact on the studied scenario, while in Section IV the braking control problem and its mathematical model are shown. Section V presents the basis of the fuzzy control approach. In Section VI, the mathematical problem formulation is depicted as a SISO nonlinear system.
with external disturbance and time-varying networked-induced delay. In Sections VII and VIII, it is presented the control synthesis for this problem. In Section IX, the validity of the proposed control design procedure is demonstrated via simulations. Section X presents some concluding remarks.

II. KHRONOSIM’S ARCHITECTURE

The developed KhronoSim’s architecture is based on scalable and distributed structure, so additional modules can be added as needed to increase the system capabilities in scenarios where a single module does not fulfill all the interface requirements. This modularity enables reuse, reconfiguration, and the ability of testing several interdependent equipment of a complex system at the same time.

At the first step of CPS development, called simulation stage, a model proposed to simulate the required system behavior is built on a computer. When it is finished and the model is found to be correct, the next step, called prototyping stage, begins and the simulated parts are incrementally replaced by the real parts or components of the cyberphysical system. The following step of the development process is called pre-production, when the system is put to work in its real physical environment until the cyberphysical system is built in its final form as it will be used and mass-produced. The reason for following these steps is that it is cheaper and quicker to change a prototype than to change the final product, and it is even cheaper and quicker to change the model. Fig. 1 depicts the integration of KhronoSim in the CPS development.

This work intends to present a purely virtual implementation (i.e., step #1) of an Antilock-Braking System over a CAN network. Thus, taking into account the KhronoSim’s architecture, it was developed two simulation modules (Fig. 2): (i) a behavioral model that encompasses the ABS, slip sensor and torque actuator models, as well as a communication network model in order to mimic the CAN network; (ii) the $H_{\infty}$ direct adaptive fuzzy controller that tracks a wheel slip reference.

III. NETWORK ENVIRONMENT

As said before, the chosen strategy to shared media access was the CAN network, which its simulation is accomplished through the TrueTime toolkit – a Matlab/Simulink-based simulator for real-time control systems and wired/wireless network communication [19], [20]. The network environment used here (Fig. 2) works in a specific fashion: the time-triggered sensor node sends information at a pre-specified time to the event-driven controller node. When the controller node receives information, starts its activity and, after a computation delay, sends the result to the actuator node. Fig. 3 illustrates the explained scenario along the timeline. Note that the total delay $\tau_d$ is the sum of all delays present in this scenario, namely: sensor data transfer time, controller data transfer time and controller computation time. Additionally, a random delay caused by packet loss may be added to $\tau_d$. In the advent of sensor packet loss, the controller will remain on idle state, and for packet loss from both nodes, the actuator will hold the last value. Table I shows the network parameters used in all the experiments that were conducted.


Table I: TrueTime’s CAN network parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor trigger sampling interval, (\tau_s)</td>
<td>6</td>
<td>[ms]</td>
</tr>
<tr>
<td>Sensor ID</td>
<td>2</td>
<td>N.A.</td>
</tr>
<tr>
<td>Controller computation delay</td>
<td>1</td>
<td>[ms]</td>
</tr>
<tr>
<td>Controller ID</td>
<td>3</td>
<td>N.A.</td>
</tr>
<tr>
<td>Data length</td>
<td>94</td>
<td>[bits]</td>
</tr>
<tr>
<td>Data rate</td>
<td>500</td>
<td>[kbps]</td>
</tr>
<tr>
<td>Loss probability</td>
<td>0.1 – 30%</td>
<td></td>
</tr>
</tbody>
</table>


Table II: Burckhardt tyre model parameters for different friction coefficients [23].

<table>
<thead>
<tr>
<th>Road conditions</th>
<th>(\eta_1)</th>
<th>(\eta_2)</th>
<th>(\eta_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry asphalt</td>
<td>1.28</td>
<td>23.99</td>
<td>0.52</td>
</tr>
<tr>
<td>Wet asphalt</td>
<td>0.86</td>
<td>33.82</td>
<td>0.3</td>
</tr>
<tr>
<td>Snow</td>
<td>0.19</td>
<td>94.13</td>
<td>0.0</td>
</tr>
</tbody>
</table>


Figure 3: Test scenario along the timeline and presentation of Sensor Trigger \(\tau_s\) and total delay \(\tau_d\).

Figure 4: Single-corner model.

IV. ANTILOCK-BRAKE SYSTEM DYNAMICS

A. Single-Corner Model

This case study is based on a model of a single wheel attached to a rigid body of mass \(m\) (Fig. 4), which represents the dynamics of a quarter of a car during straight-line braking maneuvering, and can be written as [21]

\[
\begin{align*}
J \ddot{\omega} &= R F_x - T_b, \\
\dot{m} \dot{v} &= -F_x,
\end{align*}
\]

where \(\omega\) [rad/s] is the angular speed of the wheel, \(v\) [m/s] is the longitudinal speed of the vehicle center of mass, \(T_b\) [Nm] is the braking torque, \(F_x\) [N] is the longitudinal tyre-road contact force, and \(J\) [kg m²], \(m\) [kg], and \(R\) [m] are the moment of inertia of the wheel, the single-corner mass, and the wheel radius, respectively.

B. Tyre-Road Model

Even though the most general expression of \(F_x\) depends on a large number of features of the road, tire, and suspension, it can be well approximated by

\[ F_x = F_z \cdot \mu_x(\lambda, \eta), \]

where \(F_z\) is the vertical force at the tire-road contact point, \(\mu_x(\lambda, \eta)\) is the longitudinal friction coefficient, \(\lambda\) is the longitudinal slip when braking, and \(\eta\) is a parameters vector corresponding to different road conditions. Considering the Burckhardt model [22], \(\mu_x\) can be written as

\[ \mu_x(\lambda, \eta) = \eta_1 (1 - e^{-\eta_2 \lambda}) - \eta_3 \lambda. \] (3)

By changing the values of \(\eta = (\eta_1, \eta_2, \eta_3)\), different tyre-road friction conditions can be modelled (e.g., as given in Table II).

C. Actuator Model

The braking dynamics is given by

\[
\ddot{T}_b = \frac{1}{\tau_b} \left(-T_b + \kappa_b P_b\right),
\]

where \(\kappa_b\) is the braking gain, \(\tau_b\) is the hydraulic time constant, and \(P_b\) is the braking pressure from the action of the brake pedal, which is converted to brake torque \(T_b\) and sent to the wheel dynamics. The brake torque is also subject to the constraint that \(0 < T_b < T_b^{\text{max}}\), where \(T_b^{\text{max}}\) is the maximum brake torque that the physical actuator can provide.

D. Control Objective

The longitudinal slip \(\lambda\) can be defined as the normalized relative speed between the road and the tyre, and can be written as

\[ \lambda = \frac{v - \omega R}{v} \rightarrow \dot{\lambda} = -\frac{R}{v} \omega + \frac{R \omega}{v^2} v. \] (5)

The goal of an ABS system would be to regulate \(\lambda\) to make it correspond to the peak of the current (dynamic) friction curve in order to maximize the friction between the tire and the road. In an ideal situation, the road conditions, and such friction curve characteristics, are perfectly known at each time instant. Unfortunately, estimating such a curve online from data is still an open-research problem [6]. Therefore, the reference slip is usually selected as a constant value between 0.1 and 0.2 [24].

The physical meaning of the geometric, vehicle and environmental parameters are given in Table III together with a set of
numerical values used in this work for the simulations carried out with the single-corner model depicted in this section.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>Wheel radius</td>
<td>0.33</td>
<td>[m]</td>
</tr>
<tr>
<td>( J )</td>
<td>Wheel inertia</td>
<td>1.13</td>
<td>[kg.m²]</td>
</tr>
<tr>
<td>( m )</td>
<td>Single-corner mass</td>
<td>342</td>
<td>[kg]</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravity acceleration</td>
<td>9.81</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>( v_0 )</td>
<td>Initial velocity</td>
<td>27.78</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( \tau_h )</td>
<td>Hydraulic lag</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>( \kappa_b )</td>
<td>Braking gain</td>
<td>1400</td>
<td></td>
</tr>
<tr>
<td>( T_{b_{max}} )</td>
<td>Max braking torque</td>
<td>1400</td>
<td>[N.m]</td>
</tr>
</tbody>
</table>

V. FUZZY SYSTEM

The IF-THEN rule base of a fuzzy system can be written as [25], [26]

\[ R^l : \text{IF } x_1 \text{ is } A^l_1 \text{ and } \ldots \text{ and } x_n \text{ is } A^l_n \text{ THEN } y \text{ is } G^l \text{, for } l = 1, 2, \ldots, M \] (6)

where \( x = [x_1, \ldots, x_n]^T \) and \( y \) are the input and output of the fuzzy system, respectively, \( A^l_i \) (\( i = 1, \ldots, n \)) and \( G^l \) are labels of fuzzy sets defined in \( U_i \) and \( \mathbb{R} \), respectively, \( U_i \subset \mathbb{R} \), for \( i = 1, \ldots, n \), and \( M \) is the total number of rules.

In this work, the subclass of fuzzy systems used has singleton fuzzifier, product-inference engine, and center-average defuzzifier [27], and can be written as

\[ Z(x) = \frac{1}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A^l_i}(x_i)}, \] (7)

where \( \bar{y}^l \) is the center of \( G^l \), \( A^l_i \) and \( G^l \) are the fuzzy sets in (6), and \( \mu_{A^l_i}(x_i) \) is the \( i^{th} \) input membership function in the \( l^{th} \) rule. Considering that the \( \mu_{A^l_i}(x_i) \) are fixed and \( \bar{y}^l \) are adjustable parameters, (7) can be rewritten as

\[ Z(x, \theta) = \theta^T \xi(x) = \xi(x)^T \theta, \] (8)

where \( \theta = [\theta_1, \ldots, \theta_M]^T = [\bar{y}^1, \ldots, \bar{y}^M]^T \) is the vector of adjustable parameters, and \( \xi(x) = [\xi^1(x), \ldots, \xi^M(x)]^T \) is the fuzzy basis function, where

\[ \xi^l(x) = \frac{\prod_{i=1}^{n} \mu_{A^l_i}(x_i)}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A^l_i}(x_i)}, \quad l = 1, \ldots, M. \] (9)

Theorem 1. For any given real continuous function \( \zeta(x) : U \times \mathbb{R} \) on a compact set \( U \subset \mathbb{R}^n \), and arbitrary \( \varepsilon > 0 \), there exists a fuzzy system \( Z(x) \) of the form of (8) such that \( \sup_{x \in U} |Z(x, \theta) - \zeta(x)| < \varepsilon \).

Proof 1. See [27].

VI. PROBLEM FORMULATION

Consider a class of SISO nonlinear systems of the form

\begin{align*}
\dot{x}_i &= x_{i+1}, \quad i = 1, \ldots, n-1, \\
\dot{x}_n &= f(x) + bu(t - \tau(t)) + d(t), \\
y &= x_1,
\end{align*}

(10)

where \( f \) is an unknown nonlinear continuous function, \( b \) is a positive unknown constant, \( x \triangleq [x_1, x_2, \ldots, x_n]^T = [y, \dot{y}, \ldots, y^{(n-1)}]^T \in \mathbb{R}^n \), \( u \in \mathbb{R} \), and \( y, \dot{y} \in \mathbb{R} \) are the state vector, input and output of the system, respectively, \( d(t) \in \mathbb{R} \) denotes an external disturbance, and \( \tau(t) \) represents a time-varying networked-induced delay. For simplicity \( \tau(t) \) will be written as \( \tau \).

The objective is to design an adaptive fuzzy controller, in the form of (8), such that the system state vector \( x \) follows a desired reference \( x_d \triangleq [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]^T \), under the constraint that all signals involved must be bounded and a \( H_\infty \) tracking performance must be achieved. In this work, the following analysis is based on the assumption that the system states can be measured, and \( |d(t)| \leq c_d \), where \( c_d > 0 \) is a known upper bound of \( d(t) \).

A. Filtered Tracking Error Dynamics

Let the tracking error be defined as

\[ e \triangleq x_d - x = [e(t), e, \ldots, e^{(n-1)}]^T. \] (11)

From (10), the derivative of (11) can be written as

\[ \dot{e} = Ae + B e^{(n)}, \] (12)

where

\[ e^{(n)} = x_d^{(n)} - f(x) - bu(t - \tau) - d(t), \] (13)

and

\[ A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \] (14)

The substitution of (13) into (12) gives

\[ \dot{e} = Ae + B \left( x_d^{(n)} - f(x) - bu(t - \tau) - d(t) \right). \] (15)

In [28], [29], it is introduced the finite integral of past control values defined as

\[ e_z \triangleq \int_{t-\tau}^{t} u(v) \, dv \rightarrow \dot{e}_z = u(t) - u(t - \tau), \] (15)

where \( |e_z| < e_z \) and \( e_z \geq 0 \) is the known upper bound of \( e_z \). The usefulness of \( e_z \) relies on the development of a filtered tracking error dynamics \( e_{z_{a}} \) in terms of a delay-independent control signal. Thus, let \( e_z \) be defined as

\[ e_{z_{a}} \triangleq e - B \cdot be_{z_{}} \rightarrow \dot{e}_{z_{a}} = \dot{e} - B \cdot \dot{e}_{z_{}}. \] (16)
The substitution of (14) and (15) into (16) gives
\[ \dot{e}_s = \Lambda e + B \left( x_d^{(n)} - f(x) - bu(t) - d(t) \right). \] (17)

Let the vector that describes the desired closed-loop dynamics for the error, \( k = [k_1, \ldots, k_I]^T \in \mathbb{R}^n \), be such that \( \Lambda_e \triangleq \Lambda - Bk^T \in \mathbb{R}^{n \times n} \) is a Hurwitz matrix. For any given \( Q \in \mathbb{R}^{n \times n} \), satisfying \( Q = Q^T > 0 \), a unique solution \( P \in \mathbb{R}^{n \times n} \) satisfying \( P = P^T > 0 \) exists for the following Riccati-like equation
\[ \Lambda_e^T P + PA_e = \frac{2}{r} PBB^T P + \frac{1}{\rho^2} PBB^T P = -Q, \] (18)
where \( r, \rho \in \mathbb{R}^+ \) if and only if \( 2\rho^2 \geq r \). Making \( 2\rho^2 = r \), one also obtains the following Lyapunov equation
\[ \Lambda_e^T P + PA_e = -Q. \] (19)

VII. ADAPTIVE FUZZY CONTROL APPROACH

If \( f(x) \) and \( b \) are known \textit{a priori}, then the ideal control law \( u^* \) for system (10) could be written as
\[ u^* \triangleq \frac{1}{b} \left[ -f(x) + x_d^{(n)} + k^T \xi \right]. \] (20)
As \( f(x) \) and \( b \) are unknown, (20) is not realizable. Thus, consider a certain equivalent control law with respect to (20) as follows:
\[ u \triangleq \hat{u}(x, \theta) \] (21)
and let the fuzzy control law \( \hat{u}(x|\theta) \) be in terms of (8) in order to attempt to directly approximate \( u^* \). Upon substituting (21) into (17) and after some straightforward manipulations, the closed-loop error equation becomes
\[ \dot{e}_s = \Lambda_e e_s + Bb[u^* - \hat{u}(x, \theta)] - Bd. \] (22)
Define the optimal parameter vector as
\[ \theta^* \triangleq \text{argmin}_{\theta \in \Omega_\theta} \left( \sup_{x \in \Omega_x} \left| \hat{u}(x, \theta) - u^* \right| \right), \] (23)
where \( \Omega_\theta \) and \( \Omega_x \) denote the sets of the bounds on \( \theta \) and \( x \), respectively. Define the minimum scaled-command approximation error as
\[ \omega^* \triangleq b(\hat{u}(x, \theta^*) - u^*). \] (24)
Then, the equivalent closed-loop error equation (22) can be rewritten as
\[ \dot{e}_s = \Lambda_e e_s + Bb\varphi^T \xi(x) + B\omega_d, \] (25)
where \( \varphi \triangleq \theta^* - \theta \), \( \omega_d = -\omega^* - d \), and \( \xi(x) \) is a fuzzy basis function defined by (9).

In order to determine an adjusting mechanism for \( \theta \) such that the tracking error \( e_s \) and the parameter error \( \varphi \) be minimized, consider the Lyapunov function candidate
\[ V = \frac{1}{2} e_s^T Pe_s + \frac{b}{2\gamma} \varphi^T \varphi, \] (26)
where \( \gamma \) is a positive constant, and \( P = P^T > 0 \) is a solution of (18) and (19). The time derivative of \( V \) is
\[ \dot{V} = \frac{1}{2} e_s^T Pe_s + \frac{1}{2} e_s^T Pe_s + \frac{b}{\gamma} \varphi^T \varphi + \frac{b}{2\gamma} \varphi^T \varphi. \] (27)
Using (25) and the fact that \( \varphi = -\dot{\theta} \), the above leads to
\[ \dot{V} = \frac{1}{2} e_s^T (\Lambda_e^T P + PA_e) e_s + \frac{1}{2} e_s^T (P^T B + PB) \omega_d + \frac{b}{2\gamma} \varphi^T \xi(x) \] \[ - \frac{1}{2} e_s^T Q e_s + e_s^T P B \omega_d + \frac{b}{\gamma} \varphi^T \xi(x) - \frac{b}{\gamma} \varphi^T \xi(x) \cdot 2\theta \] (28)
In order to minimize the tracking error \( e_s \) and attenuate the effect due to the approximation error \( \omega^* \) and the external disturbance \( d \) on the tracking error, \( \dot{V} \) should be negative. Therefore, a good strategy is to choose the adaptation law such that the third term in (28) is zero, as follows:
\[ \dot{\theta} = \gamma e_s^T P B \xi(x). \] (29)
Since \( Q > 0 \), then \( -\frac{1}{2} e_s^T Q e_s < 0 \). However, even assuming that \( \omega^* \) is small due to the universal approximation theorem [27], as \( \omega_d \) has the influence of \( d \) too, the term \( e_s^T P B \omega_d \) is not guaranteed to be negative, and an \( H_\infty \) tracking performance cannot be formulated.

VIII. \( H_\infty \) ADAPTIVE FUZZY CONTROL APPROACH

In order to ensure that the closed-loop system given by (10) is globally stable in the sense that the state vector \( x \) is bounded, and an \( H_\infty \) tracking performance with a prescribed disturbance attenuation level \( \rho \) is achieved, a new certain equivalent control law must be written:
\[ u = \hat{u}(x, \theta) + \frac{1}{b} u_s, \] (30)
where the fuzzy control law \( \hat{u}(x|\theta) \) is in terms of (8), and \( u_s \) is an auxiliary control term named \( H_\infty \) control and written as follows [30]:
\[ u_s = \frac{1}{r} B^T Pe_s. \] (31)
Upon substituting (30) into (17) and after some straightforward manipulation, the closed-loop error equation becomes
\[ \dot{e}_s = \Lambda_e e_s + Bb[u^* - \hat{u}(x, \theta) - \frac{1}{b} u_s] - Bd, \] \[ = \Lambda_e e_s + Bb\varphi^T \xi(x) - Bu_s + B\omega_d. \] (32)
Consider now the following Lyapunov function candidate
\[ V_e = \frac{1}{2} e_s^T Pe_s, \] (33)
where \( P \) is the same matrix used in (26). By using an adaptive law as \( \hat{u} \) as written in (29), and defining \( \alpha = B^T Pe_s / \rho \), the time derivative of \( V_e \) along (32) gives
\[ \dot{V}_e = -\frac{1}{2} e_s^T Q e_s - e_s^T PB \left( \frac{1}{r} B^T Pe_s \right) + e_s^T P B \omega_d, \] \[ = -\frac{1}{2} e_s^T Q e_s - \frac{1}{2\rho} e_s^T P B P B e_s + \omega_d^T B^T Pe_s, \] \[ = -\frac{1}{2} e_s^T Q e_s - \frac{1}{2} \left[ \alpha^2 \omega_d^2 - 2\rho \omega_d \alpha \right]. \]
\[ -\frac{1}{2} e^T Q e_s - \frac{1}{2} (\alpha - \rho \omega_d) (\alpha - \rho \omega_d) + \frac{1}{2} \rho^2 \omega_d^2 \omega_d, \]
\[ \leq -\frac{1}{2} e^T Q e_s + \frac{1}{2} \rho^2 \omega_d^2 \omega_d. \] (34)

The main result of the $H_\infty$ adaptive fuzzy control scheme is summarized in the following theorem.

**Theorem 2.** In the nonlinear system (10), if the certain equivalent control (30) is selected, then the following $H_\infty$ tracking performance with a prescribed disturbance attenuation level $\rho$ is guaranteed:

\[ \int_0^T e^T Q e_s dt \leq e^T Q e_s(0) + \rho^2 \int_0^T \omega_d^2 \omega_d dt, \]
\[ \forall T \in [0, \infty) \omega \in L_2[0, T]. \] (35)

**Proof 2.** Integrating (34) from $t = 0$ to $t = T$ yields

\[ V_e(T) - V_e(0) \leq -\frac{1}{2} \int_0^T e^T Q e_s dt + \frac{1}{2} \rho^2 \int_0^T \omega_d^2 dt. \] (36)

Since $V_e(T) \geq 0$, after a simple manipulation on the above inequality, one gets the $H_\infty$ tracking performance (35).

**A. Control Signal Boundedness**

Consider that (30) should be designed with the constraint $u \in U = [u_{\min}, u_{\max}]$, where $U$ is the command range that the physical actuator can provide, and $u_{\min} \leq u_{\max}$. To attain this goal without the need to change the design of $\hat{u}(x, \theta)$, one should dynamically saturate $u_s$ to upper and lower bounds that are dependant on $\hat{u}(x, \theta)$. Thus, $u_s$ can be substituted by

\[ \hat{u}_s = \max\{c, \min(u_s, d)\}, \] (37)

where $c = b(u_{\min} - \hat{u}(x, \theta))$, and $d = b(u_{\max} - \hat{u}(x, \theta))$.

**IX. SIMULATIONS AND RESULTS**

The objective of the $H_\infty$ AFC, presented in Section VIII, is to provide a stable control with a prescribed disturbance attenuation level $\rho$ for the ABS system discussed in Section IV, assuming that no mathematical model is available, and that there may exist unknown disturbance and unmeasured network-induced delays. In this work, the data from three sequential runs are considered for the evaluation of the controllers.

The efficiency of the proposed $H_\infty$ AFC is validated by comparison with: (a) bang-bang controller; (b) a non-adaptive basic fuzzy control (BFC) constructed as presented in Section VII but with $\gamma = 0$ (i.e. with no adaptive capabilities), and with $\theta = [-1.0 0.8 1]$ defined by trial-and-error; (c) an adaptive fuzzy control (AFC) as presented in Section VII with $\gamma = 5$ and $b = 1$; and (d) the proposed $H_\infty$ AFC as presented in Section VIII, with $\gamma = 5$, $b = 1$ and $\rho = 0.316$. In both AFC and $H_\infty$ AFC cases, it is assumed that there is no prior knowledge about the plant, and the initial parameters vector is arbitrarily set to $\theta_{ini} = [0 \ 0 \ 0]$.

**A. Performance Assessment**

For the sake of comparison of the different control strategies, their performance is assessed by the integral of square error (ISE), the variance of braking torque (var$\tau$), the stop distance ($S_d$), and the stop time ($S_t$). The obtained performance assessment results are given in Table IV.

<table>
<thead>
<tr>
<th>Run</th>
<th>ISE</th>
<th>var$\tau$</th>
<th>$S_d$ [m]</th>
<th>$S_t$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bang-Bang</td>
<td>0.011</td>
<td>2.021</td>
<td>35.90</td>
<td>2.52</td>
</tr>
<tr>
<td>BFC</td>
<td>0.011</td>
<td>0.094</td>
<td>35.97</td>
<td>2.49</td>
</tr>
<tr>
<td>AFC</td>
<td>0.048</td>
<td>0.001</td>
<td>58.36</td>
<td>3.43</td>
</tr>
<tr>
<td>$H_\infty$ AFC</td>
<td>0.026</td>
<td>0.050</td>
<td>41.71</td>
<td>2.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>ISE</th>
<th>var$\tau$</th>
<th>$S_d$ [m]</th>
<th>$S_t$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bang-Bang</td>
<td>0.011</td>
<td>2.021</td>
<td>35.90</td>
<td>2.52</td>
</tr>
<tr>
<td>BFC</td>
<td>0.011</td>
<td>0.094</td>
<td>35.97</td>
<td>2.49</td>
</tr>
<tr>
<td>AFC</td>
<td>0.008</td>
<td>0.097</td>
<td>35.85</td>
<td>2.47</td>
</tr>
<tr>
<td>$H_\infty$ AFC</td>
<td>0.007</td>
<td>0.097</td>
<td>35.65</td>
<td>2.46</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>ISE</th>
<th>var$\tau$</th>
<th>$S_d$ [m]</th>
<th>$S_t$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bang-Bang</td>
<td>0.011</td>
<td>2.021</td>
<td>35.90</td>
<td>2.52</td>
</tr>
<tr>
<td>BFC</td>
<td>0.011</td>
<td>0.094</td>
<td>35.97</td>
<td>2.49</td>
</tr>
<tr>
<td>AFC</td>
<td>0.003</td>
<td>0.099</td>
<td>34.95</td>
<td>2.42</td>
</tr>
<tr>
<td>$H_\infty$ AFC</td>
<td>0.003</td>
<td>0.105</td>
<td>34.95</td>
<td>2.42</td>
</tr>
</tbody>
</table>

The bang-bang approach abruptly switches the control input between 0 and 1 during the braking phase. As a result, the driver may experience jerks, which cause discomfort. In addition, such a control may reduce the life of the actuator. The BFC approach has a considerably less braking torque activity. However, its parameter vector $\theta$ is found through trial-and-error experiments, and its value is constant. The AFC approach does not require prior knowledge about the plant and guarantees stability. However, it relies on the hope that the $\omega_d$ in (25) is small and, at the first run, the controller cannot have a good performance because it did not have enough time to adapt to the system. Some temporal responses are shown in Figs. 5, 6, and 7. They depict the longitudinal slip $\lambda$ (5), linear velocity of the car and wheel (5), brake torque $T_b$ (4), and actual control output (red) on the Run 1 of the Bang-Bang approach, as well as on Runs 1 and 3 of the proposed HAFC approach, respectively.

Through these results, even if no plant knowledge is available, and there are networked-induced delays, the robust and adaptive characteristics of the controller proposed in this work has shown excellent performance, softer breaking, and softer actuator effort, especially when compared with the bang-bang approach, which is still currently available in today’s cars [31].
X. CONCLUSION

This work presented a proof of concept of a cyberphysical system test framework, called KhronoSim, by showing a purely virtual implementation of an Antilock-Braking System over a CAN network.

As a case study, it was considered an $H_{\infty}$ adaptive fuzzy approach as a wheel slip controller in order to tackle the time-varying networked-induced delay problem in addition to the nonlinearities of the plant. The simulation results demonstrated that the system output converges to the reference signal and all the signals of the closed loop systems are bounded, i.e., closed loop system is guaranteed stable, and reveal the effectiveness of the proposed controller through comparative results.

ACKNOWLEDGMENT

This work was supported by Project KhronoSim “System for Simulation and Test of Complex Systems” (reference: KhronoSim/2016/17611), co-financed by the “Competitiveness and Internationalization Operational Programme” (COMPETE 2020), Portugal 2020 (PT2020), and by the European Union through the European Regional Development Fund (ERDF), and by Critical Software S.A.. Jérôme Mendes has been supported by Fundação para a Ciência e a Tecnologia (FCT) under grant SFRH/BPD/99708/2014.

REFERENCES


