Robot Comanipulation with Active Impedance Control

Rui Cortesão¹, Brian Zenovich², Rui Araújo¹, and William Townsend²

¹University of Coimbra, Institute of Systems and Robotics, 3030 Coimbra, Portugal
²Barrett Technology Inc., Massachusetts, 02138-4555 USA
emails: cortesao@isr.uc.pt, bz@barrett.com, rui@isr.uc.pt, wt@barrett.com

Abstract—The paper presents active impedance control for robotic comanipulation tasks, enabling virtual contact interactions. Computed torque control in the task space powered by multiple-output active observers (AOBs) is proposed, enhancing haptic perception. Forces and force derivatives are artificially measured from position data around an equilibrium point that can move with time. Control techniques to deal with critical impedances are introduced, taking into account the noise distribution along the system. Stochastic design is discussed. A dynamic model of the redundant lightweight 7-DOF WAM™ arm is derived and evaluated, playing a key role in the control design. Experiments for small and high impedances are presented, highlighting merits and limitations of the approach. A comparative study between active and non-active impedance control is made.

I. INTRODUCTION

Robotic manipulation with haptic feedback can be divided into two major groups: telemanipulation and comanipulation. In the first group, the human operator commands the robotic arm without physical contact, and in the second one the operator grabs directly the robot to perform a task. Explicit force sensing is not mandatory to achieve haptic feedback, which can be provided by position errors through virtual coupling [6]. Moreover, virtual haptic walls with variable compliance can be added to the task scenario not only to restrict the workspace but also to ensure safe manipulation. Overlaying and calibrating such walls on top of geometric information (e.g. vision) can greatly improve manipulation skills, being particularly useful for critical and delicate tasks. Comanipulation has several paradigms [10], such as: 1) Degrees of freedom (DOF) sharing. For a redundant arm with respect to the robot task space, the operator and robot independently control some DOF. 2) Space sharing. Haptic constraints to avoid forbidden zones, signaling dangerous workspaces. 3) Motion filtering. Equivalent to "frequency domain sharing", where commanded human motions are properly filtered by the control design. 4) Gravity compensation. The robot only compensates for the robot mass, including its tool. Therefore, the human operator feels a free-floating device. 5) Active guidance. The robot applies active (by opposition to resistive) forces to indicate to the operator desired motion directions. 6) Active impedance. This paradigm is discussed in this paper. A desired robot impedance is specified in task space, actively controlled by AOB design. Explicit force sensing is not required.

In general, comanipulation can resort from measured and computed forces to accomplish a desired interaction behavior. Force sensors are usually attached to the end-effector to provide high sensitivity and resolution to tool interactions. Artificial forces can be computed and applied to the robot, taking into account posture and task constraints, as well as human-robot-environment interactions. Comanipulation architectures are excellent candidates for human-robot skill transfer, since part of the robot motion is based on human skills. Such skills can be extracted through sensor-based learning to boost intelligent behaviors [5].

The paper is organized as follows. Section III discusses active impedance control with AOBs. The control architecture is introduced, taking into account computed torque techniques, task space formulation and AOB design. Dynamic model assessment and comanipulation experiments are presented in Section IV. Section V concludes the paper.

II. RELATED WORK

Comanipulation techniques can be divided into two major classes, reflecting the presence or absence of force sensors in the loop. There is a growing interest of such techniques in the medical field, particularly for surgical robotics, rehabilitation robotics and assistive technologies, where a lot of applications demand direct human-robot interaction. Several force control algorithms have been analyzed in [11] for comanipulation, where adaptive force control techniques are addressed. In the medical field, Taylor and co-authors presented micro-surgical augmentation with small interaction forces in [13]. In [9], a comanipulation system without force sensor is presented, where there is only one DOF left to perform needle insertion. More recently, in [14] force control for kinematically constrained manipulators is analyzed in the context of minimally invasive surgery. None of these techniques have used model reference adaptive control (MRAC) strategies to impose desired impedance behaviors. In our control scheme with AOBs, MRAC is tuned by stochastic parameters, which is not the approach of classical MRAC techniques, leaving space to shape impedances with control techniques.

III. ACTIVE IMPEDANCE CONTROL

According to Hogan [7], the idea of impedance control is to assign a prescribed robot dynamic behavior in the presence of external interactions, matching the dynamics of mass-spring-damper systems. The end-effector velocity $\dot{X}$ and the interaction force $F_i$ are related by a mechanical impedance...
In terms of position,
\[ F_h(s) = sZ(s)X(s), \]  
(2)

where
\[ sZ(s) = As^2 + Bs + K \]
(3)

and \( A, K \) and \( B \) are the parameters of a mass-spring-damper system, respectively. Equation (3) is based on physical systems, having no active components. Therefore, (3) can be generalized to
\[ sZ(s) = \sum_{i=-n_1}^{n_2} \nu_is^i, \]
(4)

with \( n_1 \geq 0, n_2 \geq 0, \) and \( \nu_i \) impedance parameters. If \( n_1 > 0, \) (4) represents an active impedance system, due to integral actions on \( X(s). \)

A. Robot Dynamics in Task Space

Given a set of generalized coordinates \( q \) (usually, joint angles for revolute joints) describing robot’s posture, the well-known robot dynamics is given by
\[ M(q)\ddot{q} + v(q, \dot{q}) + g(q) = \tau \]
(5)

where \( M(q) \) is the mass matrix, \( v(q, \dot{q}) \) is the vector of Coriolis and centripetal forces, \( g(q) \) is the gravity term, and \( \tau \) is the generalized torque acting on \( q. \) Making the pre-compensation of \( v(q, \dot{q}) \) and \( g(q), \)
\[ M(q)\ddot{q} = \tau', \]
(6)

where \(^1\)
\[ \tau = \tau' + \dot{v}(q, \dot{q}) + \dot{g}(q). \]
(7)

Equation (6) can be represented in task space by
\[ \Lambda_t(q)\ddot{X}_t - \Lambda_t(q)\dot{J}_t(q)\dot{q} = F_t, \]
(8)

with
\[ \dot{X}_t = J_t(q)\dot{q}, \]
(9)

\[ \Lambda_t^{-1}(q) = J_t(q)M^{-1}(q)J_t^T(q), \]
(10)

and
\[ \tau_t = J_t^T(q)F_t. \]
(11)

\( \Lambda_t, X_t, J_t, F_t \) and \( \tau_t \) are respectively the operational space mass matrix, Cartesian position, Jacobian matrix, Cartesian force and task torque. \( F_t \) includes all Cartesian forces referred to the end-effector, such as computer commanded, friction, coupling, human and environment forces. \( J_t \) is a truncated non-squared matrix \((3 \times 7), \) since our task space is the 3D Cartesian space.

\(^1\)The symbol “ means estimate.

B. Feedback Linearization

An active impedance controller can be designed around the equilibrium point generating reactions to external forces. Without disturbance forces\(^2\), \( F_t \) in (8) is equal to the commanded force \( F_{c,t}. \) It is computed by
\[ F_{c,t} = -\dot{\Lambda}_t(q)\dot{J}_t(q)\dot{q} + \dot{\Lambda}_tf_t^*, \]
(12)

to achieve the desired decoupled system
\[ \ddot{X}_t = f_t^*. \]
(13)

Equation (13) represents the dynamics of a unitary mass for each Cartesian dimension. \( f_t^* \) is an acceleration (see (12)), being an input parameter. Inserting a desired Cartesian stiffness \( K_{s,n} \) necessary to compute a virtual force from position displacements\(^3\), and taking the system delay \( T_d \) into account (mainly due to signal processing), as well as a damping factor \( K_2 \) (explained in the sequel), the overall system plant \( G(s) \) represented in Figure 1 is obtained. Since \( T_d \) is small, for each Cartesian dimension
\[ G(s) \approx \frac{K_{s,n}e^{-sT_d}}{s(s + K_2)}. \]
(14)

Its equivalent temporal representation is
\[ \ddot{y}(t) + K_2\dot{y}(t) = K_{s,n}u(t - T_d), \]
(15)

where \( y(t) \) is the computed Cartesian force, based on position displacements around the equilibrium point, and \( u(t) \) is the plant input. Defining the state variables \( x_1(t) = y(t) \) and \( x_2(t) = \dot{y}(t), (15) \) can be written as
\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & -K_2
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
K_{s,n}
\end{bmatrix} u(t - T_d).
\]
(16)

In compact form,
\[
\begin{cases}
\dot{x}(t) = Ax(t) + Bu(t - T_d) \\
y(t) = Cx(t)
\end{cases}.
\]
(17)

\(^2\)In a real scenario there are disturbance forces due to modeling errors. However, the control architecture needs to know the desired model, which is disturbance free, to generate proper model reference adaptive control actions.

\(^3\)For comanipulation in contact, \( K_{s,n} \) should not be too small, compared to the system stiffness. See [6] for stability and robustness analysis under contact.
Discretizing (17) with sampling time $h$, the equivalent discrete time system is of form\(^4\)

\[
\begin{aligned}
    x_{r,k} &= \Phi_r x_{r,k-1} + \Gamma_r u_{k-1} \\
    y_k &= C_r x_{r,k}
\end{aligned}
\]  

(18)

In our case, $x_k$ has dimension three. The first two states of the discretized system are the measured ones, representing the end-effector force and its derivative, respectively. The other state appears due to $T_d$, being equal to $u_{k-1}$, the discrete version of $u(t-T_d)$.

C. AOB Design for Active Impedance Control

1) AOB Concept: To accomplish model-reference adaptive control, the AOB reformulates the Kalman filter, based on [6, 4]:

- A desired closed loop system (reference model) that enters in the state estimation.
- An active state $p_k$ (extra state) to describe an equivalent disturbance referred to the system input.
- The stochastic design of the Kalman matrices for the AOB application.

2) AOB Control Scheme: Fig. 2 represents the AOB control scheme for comanipulation with active impedance control. The value $F_{i,k}$ represents the desired force at equilibrium. Changing $F_{i,k}$, the equilibrium point moves. The motion dynamics due to $F_{i,k}$, and the active impedance during comanipulation depend on the closed loop poles. The measurement noise of the Cartesian position $\eta_x$, affects the control design, being the major limitation to achieve high impedance behaviors. Theoretically, the damping feedback through $K_s$ can be implemented through the state feedback gain $L_a$. However, this solution is more noisy if $K_s$ is high, due to the noise amplification in $\hat{F}_X$, through $K_{s,n}$. Another solution would be to reformulate the state space variables to accommodate $X$ instead of $\hat{F}_X$, (not addressed in this paper).

With the AOB in the loop, the overall system can be represented by [6]

\[
\begin{bmatrix}
    x_{r,k} \\
p_k
\end{bmatrix} = \begin{bmatrix}
    \Phi_r & \Gamma_r \\
0 & 1
\end{bmatrix} \begin{bmatrix}
    x_{r,k-1} \\
p_{k-1}
\end{bmatrix} + \begin{bmatrix}
    \Gamma_r \\
0
\end{bmatrix} u_{k-1} + \xi_k
\]

(19)

and

\[
y_k = C_a \begin{bmatrix}
x_{r,k-1} \\
p_{k-1}
\end{bmatrix}^T + \eta_k,
\]

(20)

where

\[
u_{k-1} = r_{k-1} - [L_r \ 1] \begin{bmatrix}
x_{r,k-1} \\
p_{k-1}
\end{bmatrix}.
\]

(21)

The state $x_{r,k}$ is

\[
x_{r,k} = \begin{bmatrix}
F_{X_i,k} & \hat{F}_{X_i,k} & u_{k-1}
\end{bmatrix}^T,
\]

(22)

where $F_{X_i,k}$ and $\hat{F}_{X_i,k}$ are the discrete versions of $F_X$ and $\hat{F}_X$, respectively. The stochastic inputs $\xi_k = [\xi_{x_{r,k}} \ \xi_{p_k}]^T$ and $\eta_k = [\eta_{F_X} \ \eta_{F_{\hat{F}_X}}]^T$ represent respectively model and measurement uncertainties. Since two outputs are measured,

\[
C_a = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}.
\]

(23)

The state estimate of (19) is

\[
\hat{x}_{r,k} = \begin{bmatrix}
\Phi_r - \Gamma_r L_r \\
0 & 1
\end{bmatrix} \hat{x}_{r,k-1} + \begin{bmatrix}
\Gamma_r \\
0
\end{bmatrix} u_{k-1} + K_k (y_k - \hat{y}_k),
\]

(24)

with

\[
\hat{y}_k = C_a \left(\begin{bmatrix}
\Phi_r - \Gamma_r L_r \\
0 & 1
\end{bmatrix} \hat{x}_{r,k-1} \right) + \begin{bmatrix}
\Gamma_r \\
0
\end{bmatrix} u_{k-1} + K_k (y_k - \hat{y}_k),
\]

(25)

and

\[
y_k = \begin{bmatrix}
F_{X_i,k} & \hat{F}_{X_i,k}
\end{bmatrix}^T.
\]

(26)

The Kalman gain $K_k$ reflects the uncertainty associated to each state, which is a function of $\xi_k$ and $\eta_k$ [3, 8]. It is given by

\[
K_k = P_{ik} C_a^T \left[C_a P_{ik} C_a^T + R_k\right]^{-1},
\]

(27)

\[
\text{with}
\]

\[
P_{ik} = \Phi_n P_{k-1} \Phi_n^T + Q_k
\]

(28)

\[
\text{and}
\]

\[
P_k = P_{ik} - K_k C_a P_{ik}.
\]

(29)

$\Phi_n$ is the augmented open loop matrix,

\[
\Phi_n = \begin{bmatrix}
\Phi_r & \Gamma_r \\
0 & 1
\end{bmatrix}.
\]

(30)

The system noise matrix $Q_k$ is

\[
Q_k = \begin{bmatrix}
Q_{x_{r,k}} & 0 \\
0 & Q_{p_k}
\end{bmatrix}.
\]

(31)

$P_k$ and $R_k$ are respectively the mean square error and measurement noise matrices.

3) Stochastic Design: The stochastic parameters of the AOB have to be designed for the comanipulation context. The noise associated with the measures is correlated. The measured force can be written as

\[
F_{X_i,k} = F_{s,k} + \eta_{F_{\hat{F}_X}},
\]

(32)

where $F_{s,k}$ is the signal corrupted by noise $\eta_{F_{\hat{F}_X}}$. In the same way, $\hat{F}_{X_i,k}$ is

\[
\hat{F}_k = \frac{F_{r,k} - F_{s,k-1}}{h} + \eta_{\hat{F}_k},
\]

(33)

with $\eta_{\hat{F}_k}$ given by

\[
\eta_{\hat{F}_k} = \frac{\eta_{F_{\hat{F}_X}} - \eta_{F_{\hat{F}_X}}}{h}.
\]

(34)

Since

\[
R_k = E\left\{\left[\begin{array}{c}
\eta_{F_{\hat{F}_X}}^2 \\
\eta_{F_{\hat{F}_X}} \eta_{\hat{F}_k}
\end{array}\right]\right\},
\]

(35)
straightforward analysis of (34) and (35) gives\(^6\)

\[
R_k = E \{ \eta_F^T \} \begin{bmatrix} 1 & 1 & 1 & 1/h & 1/h^2 \end{bmatrix} . \tag{36}
\]

The Kalman gains only depend on the relations between \(R_k\) and \(Q_k\) \cite{4}. Hence, a normalized value for \(R_k\), \(\hat{R}_k\), has been used in the design. From (36),

\[
\hat{R}_k = \begin{bmatrix} 1 & 1 & 1 & 1/h & 1/h^2 \end{bmatrix} . \tag{37}
\]

A sensor-based estimation strategy is adequate for the measured states, enabling to accurately track comanipulation forces. Therefore, the uncertainty associated with those states, coded in \(Q_k\), is relatively high, making the estimates more sensitive to measures. The scaling factor of stochastic parameters should make observer dynamics faster than system dynamics. In the design,

\[
Q_k = \begin{bmatrix} 10^{-1} & 0 & 0 & 0 & 0 \\ 0 & 10^{-1} & 0 & 0 & 0 \\ 0 & 0 & 10^{-12} & 0 & 0 \\ 0 & 0 & 0 & 10^{-2} \end{bmatrix} . \tag{38}
\]

The Kalman gains \(K_k\) are not only a function of \(R_k\) and \(Q_k\), but also of the system plant, which includes the control parameters \(K_2\) and \(K_{s,n}\) (derived in the sequel). The stochastic parameters (e.g., \(K_k\), \(R_k\) and \(Q_k\)) and the control gains (e.g., \(L_a\) and \(K_2\)) are all based on SI units.

4) Control Design: The feedback gains have to be properly designed based on the desired active impedance for comanipulation. The critical scenario happens for high values of \(K_{s,n}\), necessary for comanipulation on hard surfaces. In this case, the derivative gain should come mainly from \(K_2\), and not from the force derivative feedback. A virtual PD controller (\(K_1\) and \(K_2\) gains) can be designed to achieve a desired closed loop response. This procedure assigns the \(K_2\) gain\(^7\). The same desired response\(^8\) is used to compute the state feedback gain, taking also into account the deadtime and discretization procedures. In this way, the state feedback gain for the derivative term (\(L_2\)) will be very small, reducing the effect of noise in the system. Moreover, \(L_1\) plays the role of \(K_1\) referred to the force data,

\[
L_1 = \frac{K_1}{K_{s,n}} . \tag{39}
\]

In the presence of force sensors, the feedback from \(L_1\) instead of \(K_1\) disables position-controlled behaviors. For moderate or small values of \(K_{s,n}\), the feedback distribution of the derivative terms (the critical ones) can be balanced between \(K_2\) and the state feedback. This analysis does not take into account signal to noise ratios but only noise values, since the noise distribution always exists even without signal inputs (i.e., zero values for the signal).

D. Active Impedance Perception

Without loss of generality, let’s consider the equilibrium point at \(F_{r,k} = 0\) and \(X_t = 0\) (see Fig. 2). When the human operator applies an external force \(F_h\) to the robot, the controller reacts through

\[
f_t^* = -K_2 \dot{X}_t - L_a \begin{bmatrix} \ddot{X}_t \\ \ddot{F}_X \end{bmatrix} \hat{F}_h(t) \left( t - T_d \right) \tag{40}
\]

where \(\hat{p}(t)\) is the continuous version of \(\tilde{p}_h\). From the stochastic design (sensor-based strategies), and knowing that the term \(L_2 \dot{u}(t - T_d)\) has negligible influence in \(f_t^*\),

\[
f_t^* \approx -(K_2 + L_2 K_{s,n}) \dot{X}_t - L_1 K_{s,n} X_t - p(t) . \tag{41}
\]

Therefore, from (8) and (12),

\[
\lambda_t \begin{bmatrix} \ddot{X}_t + (K_2 + L_2 K_{s,n}) \dot{X}_t + L_1 K_{s,n} X_t + p(t) \end{bmatrix} \approx F_h . \tag{42}
\]

Without considering the influence of \(p(t)\), (42) corresponds to the classical impedance given by (2) and (3). The gains due

\(^6\)The \(\eta_{F_t}\) noise is not white, since it correlates with the previous sample. It is approximated by white noise within the system bandwidth (Kalman design).

\(^7\)The \(K_1\) gain is never used in the design.

\(^8\)In the design we have used a critically damped response with time constant \(\tau_f\).
to velocity and position feedback affect respectively damping and spring parameters. The mass matrix \( \Lambda_t \) shapes all impedance parameters, generating couplings among Cartesian axes. When the off-diagonal terms of \( \Lambda_t \) are small compared to the main diagonal ones, a good decoupling behavior is accomplished\(^9\). Nevertheless, an external force \( F_{\text{ext}} \) can be added to \( F_h \) to compensate couplings in (42). \( F_{\text{ext}} \) is given by

\[
F_{\text{ext}} = \hat{\Lambda}_t^d \left[ \ddot{X}_t + (K_2 + L_2 K_{s,n}) \dot{X}_t + L_1 K_{s,n} X_t + p(t) \right].
\]  

(43)

The matrix \( \hat{\Lambda}_t^d \) corresponds to \( \hat{\Lambda}_t \) with the main diagonal set to zero. Equation (42) becomes then decoupled for each task dimension. Let’s analyze now the role of \( p(t) \). In discrete terms,

\[
\hat{p}_k = \hat{p}_{k-1} + K_k \left[ (F_{X_t,k} - \hat{F}_{X_t,k}) \right] T.
\]

(44)

From (24), (25) and \( r_{k-1} = 0 \),

\[
\begin{bmatrix}
\dot{x}_{r,k} \\
\hat{p}_k
\end{bmatrix} = \sum_{j=1}^{k} \Phi_e^{-j} K_j \begin{bmatrix}
F_{X_t,k} \\
F_{X_t,k}
\end{bmatrix},
\]

(45)

with

\[
\Phi_e = [I - K_k C_a] \begin{bmatrix}
\Phi_r - 1 & T \phi_r & L_r & 0 & 1
\end{bmatrix}.
\]

(46)

It can be inferred from (45) that the state estimate is a linear function of all past measured variables \( F_{X_t,k} \) and \( F_{X_t,k} \). Therefore, (44) performs integral actions on \( X_{t,k} \) and \( \dot{X}_{t,k} \), making (42) an active impedance system. From (7), (11), (12), (40) and (43) the commanded torque for the task \( \tau_{r,t} \) is

\[
\tau_{r,t} = \hat{g}(q) + \hat{v}(q, \dot{q}) - J_r^T \hat{\lambda}_t \dot{q} + J_r^T \hat{\lambda}_t \dot{q} + J_r^T F_{\text{ext}},
\]

(47)

Each component of \( f^*_r \) is independently computed by

\[
f^*_r = r_k - L_o \left[ x_{r,k} \hat{p}_k \right] T - K_2 \dot{X}_t,
\]

(48)

where the control gains (i.e the active impedance parameters) may vary for each task dimension.

IV. EXPERIMENTS

This section analyses the quality of the 7-DOF WAM dynamic model, followed by manipulation experiments.

A. Experimental Setup

A picture of the lightweight 7-DOF WAM is depicted in Fig. 3. The robot, joint-torque controlled, is connected by CAN to a shuttle PC (AMD Duron Applebeed x86 processor at 1.8 [GHz]) running RTAI Linux. The roundtrip communication time is about 0.8 [ms]. The control sampling time was set to \( h = 1.4 \) [ms]. The overall system deadtime \( T_d = h \).

\(^9\)For lightweight robots, the \( \Lambda_t \) values are small.

\(^{10}\)There is a linear relation between \( F_{X_t,k} \), \( F_{X_t,k} \), and \( X_{t,k} \) and \( \dot{X}_{t,k} \).

B. Dynamic Model of the 7-DOF WAM

The dynamic model of (5) can be computed either by recursive Newton-Euler (RNE), Euler-Lagrange (EL) or a combination of both methods. In our setup, \( M(q) \) is based on EL [12]. The terms \( v(q, \dot{q}) \) and \( g(q) \) come from RNE\(^{11}\). The value of \( M(q) \) is given by

\[
M(q) = \sum_{i=1}^{n} \left[ \begin{array}{c}
\mu_i J_{v_i}(q) \\
\mu_i J_{v_i}(q)
\end{array} \right] (49)
\]

\[
+ \mu_i J_{w_i}(q)^T R_i(q) I R_i(q)^T J_{w_i}(q),
\]

(49)

where \( \mu_i \), \( J_{v_i}(q) \), \( J_{w_i}(q) \) and \( R_i(q) \) are respectively the mass of each link, the Jacobians associated with linear and angular velocities, the rotation matrix in base coordinates and the inertia tensor in link coordinates. All these quantities are referred to the center of mass of link \( i \). For the 7-DOF WAM arm, the values of \( \mu_i \) and \( R_i \) obtained with SolidWorks\textsuperscript{TM} are in Barrett’s manual [2], which take into account specific characteristics of the WAM design. \( J_{v_i}(q) \) and \( J_{w_i}(q) \) can be numerically computed at each time step by

\[
\begin{bmatrix}
J_{v_i}(q) \\
J_{w_i}(q)
\end{bmatrix} = \begin{bmatrix}
a_0 & \ldots & a_{i-1} & 0 & \ldots & 0 \\
0 & \ldots & z_{i-1} & 0 & \ldots & 0
\end{bmatrix}
\]

(50)

with

\[
a_k = z_k \times (o_{v_i} - o_k) \quad k = 1, \ldots, i - 1.
\]

(51)

\( o_{v_i} \) is the center of mass of link \( i \), and \( o_k \) is the origin of link frame \( k \), i.e.

\[
o_k = \begin{bmatrix}
x_k \\
y_k \\
z_k
\end{bmatrix} T.
\]

(52)

Using this procedure, the computation of \( M(q) \) in the shuttle PC takes\(^{13} \) 38 [\mu s]. With RNE, the computation times of \( M(q) \), \( v(q, \dot{q}) \) and \( g(q) \) are respectively 13 [\mu s], 2 [\mu s] and below 1 [\mu s]. For manipulation techniques that require the partial Jacobians \( J_{v_i}(q) \) and \( J_{w_i}(q) \), there might be an advantage of using (49), instead of RNE methods.

\(^{11}\)Setting \( \dot{q} = 0, v(q, \dot{q}) + g(q) = \tau \). Additionally, \( g(q) = \tau \) for \( \dot{q} = 0 \) and \( v(q, \dot{q}) \) can be written in the form \( C \dot{q}(q, \dot{q}) \).

\(^{12}\)Each column of \( M(q) \) can also be computed by RNE techniques. For example, the first column of \( M(q) \) is the value of \( \tau \) for \( \dot{q} = [1 0 0 \ldots 0] \) and \( \dot{q} = 0 \) subtracted by \( g(q) \).

\(^{13}\)Computation times vary, depending on compilation flags, hardware, and coding techniques. We were able to achieve 20 [\mu s] for (49) with a recent PC.
C. Dynamic Model Assessment

To assess the dynamic model, (12) is applied to the robot with \( f^*_x \) equal to a step function starting at \( T_0 \),

\[
f^*_x = A_f \text{Step}(t - T_0). \tag{53}
\]

Perfect modeling will make each Cartesian position follow a double integrator response (see (13)),

\[
X_t = A_f \frac{(t - T_0)^2}{2}. \tag{54}
\]

Fig. 4 presents results. \( A_f = 10 \text{ [m/s}^2\text{]} \), and a resistive (by opposition to commanded) acceleration due to friction of 7 [m/s^2] acting against \( f^*_x \) was considered to plot the desired responses. From Fig. 4 it can be inferred that at the beginning of the step, the dynamic model tracks well the real response and couplings between axis are small. Due to the open-loop nature of this test, a poor matching performance as time goes by was expected.

D. Comanipulation Experiments

The comanipulation control design follows the procedure of Sections III-C.3 and III-C.4, respectively. This section reports control results for small and high impedances. A comparative study is also made, showing differences between classical and active impedances.

1) Design of Small and High Active Impedances: Setting \( K_{x,n} = 50000 \text{ [N/m]} \), critically damped responses with different time constants \( \tau_f \) entail different active impedances. For small impedance, \( \tau_f = 0.2 \text{ [s]} \), giving

\[
K_2 = 10.0, \tag{55}
\]

\[
L_a \approx \begin{bmatrix} 5 \times 10^{-4} & 1.04 \times 10^{-6} & 4.87 \times 10^{-5} & 1 \end{bmatrix}, \tag{56}
\]

and

\[
K_k \approx \begin{bmatrix} 0.5254 & -0.0003 \\ 64.3958 & -0.0272 \\ 0.0940 & -0.0000 \\ 0.0940 & -0.0000 \end{bmatrix}. \tag{57}
\]

For high impedance, \( \tau_f = 0.03 \text{ [s]} \), with

\[
K_2 \approx 66.6667, \tag{58}
\]

\[
L_a \approx \begin{bmatrix} 2.22 \times 10^{-2} & 4.36 \times 10^{-5} & 2.1 \times 10^{-3} & 1 \end{bmatrix}, \tag{59}
\]

and

\[
K_k \approx \begin{bmatrix} 0.4833 & -0.0003 \\ 45.6231 & -0.0202 \\ 0.0985 & -0.0000 \\ 0.0985 & -0.0000 \end{bmatrix}. \tag{60}
\]

Smaller time constants generate higher feedback gains, increasing the system impedance (see (42)).

Fig. 5 presents comanipulation results. In Fig. 5.a the human operator is able to move the arm a couple of centimeters, feeling soft impedance around the equilibrium point. In Fig. 5.b the arm only moves a few millimeters. In both situations, \( f^*_x \) is mainly due to \( \hat{p}_k \). For the high impedance case (Fig.
control parameters were integral actions (see Figure 6 around 85 [s]). The main does not go to the equilibrium point due to the lack of position are degraded with respect to the active impedance with non-active impedance. The relations between force and sure human forces. Figure 6 presents comanipulation results with active impedance control. The same controller is kept for both cases, eliminating only active state actions in the classical impedance control scheme proposed in this paper with classi-

5. b) there are however more high frequency components in \( f_2^* \) due to the \( K_2 \) gain that amplifies the Cartesian position noise.

2) Comparative Study: This section compares the active impedance control scheme proposed in this paper with classical impedance control. The same controller is kept for both cases, eliminating only active state actions in the classical control.

A force sensor was attached to the end-effector to measure human forces. Figure 6 presents comanipulation results with non-active impedance. The relations between force and position are degraded with respect to the active impedance case (see Figure 7). After comanipulation, the robot position does not go to the equilibrium point due to the lack of integral actions (see Figure 6 around 85 [s]). The main control parameters were \( K_{x,n} = 5000 \) [N/m], \( K_2 = 6.66 \) and

\[
 L_a \approx \begin{bmatrix} 0.468 & 0.018 & 0.125 & 1 \end{bmatrix}. \tag{61}
\]

V. CONCLUSIONS

The concept of active impedance has been introduced. It extends the formulation of mechanical impedance to accommodate integral actions between applied force and measured position. A multiple output active observer has been designed on top of operational space techniques to achieve active impedance control for comanipulation. The proposed paradigm does not include force sensors in the loop. Virtual forces are computed from position displacements around an equilibrium point. Control techniques that address the distribution of feedback gains based on measurement noise have been introduced, as well as the stochastic design for comanipulation. Noise amplification by derivative terms are the practical limitation for very stiff behaviors. The active impedance parameters are shaped by the operational space mass matrix, control gains and stochastic gains. Impedance decoupling techniques have been introduced, enabling decentralized design for each Cartesian dimension. Experimental results have validated the approach for small and high impedances. A comparative study between active and non-active impedance control for comanipulation has been presented, highlighting the merits of the approach. Moreover, a dynamic model of the 7-DOF WAM robot has been tested in task space to assess not only the quality of feedback linearization techniques but also control robustness to modeling errors.

VI. ACKNOWLEDGMENTS

This work was supported in part by the Portuguese Science and Technology Foundation (FCT) Project PTDC/EEA-ACR/72253/2006.

REFERENCES


